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MODEL SPECIFICATION AND ESTIMATION EFFECTS IN APPLIED DEMAND ANALYSIS USING MICRODATA

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Model Specification and Estimation Effects in Applied Demand Analysis Using Microdata[1]

by

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Abstract

A rank three demand system incorporating labour force participation, non-separability of demands from excluded goods and non-exact aggregation in income and household characteristics is estimated using Canadian microdata. Various models are estimated using nonlinear three-stage least squares (NL3SLS) and maximum likelihood (ML) methods.

It is found that demands are not separable from labour force variables or other goods, and that exact aggregation is rejected. The rank three requirement does not seem important here, however. Predictive power is extremely poor for NL3SLS relative to ML estimated models, even though some variables appear to be non-exogenous.

Keywords:

exact aggregation; separability; rank three demands; labour supply; microdata.

1 Introduction

The availability of large cross-sections of household expenditure microdata has given increased impetus to studies in applied demand analysis. These data permit modelling detailed household characteristics effects, empirical analyses of the conditions under which micro parameters can be recovered using aggregate data, estimation of household equivalence scales, examination of the importance of labour force variables as conditioning variables in demand models, and the need to generalise the model specifications used to rank three systems.

Examples of this recent research include the work of Blundell, Pashardes and Weber (1983), who find that aggregate data alone are unable to yield reliable estimates of structural price and income coefficients. This is demonstrated in the context of a demographically flexible model, using British Family Expenditure Survey (FES) data. They also find, however, that aggregate data can still provide reasonable forecasts, so long as these models contain aggregation factors, trend and seasonal components.

Conditions for the identifiability of general household equivalence scales have been explored by Lewbel (1989) and by Blundell and Lewbel (1991). The latter study also presented estimates of relative general equivalence scales, as did Pashardes (1991). Deaton and Muellbauer (1986), Deaton, Ruiz-Castillo and Thomas (1989), Dickens, Fry and Pashardes (1993) and Nicol (1994) also provide some examples of theoretical and empirical analyses of equivalence scales, and discuss the inherent difficulties in estimating these.

The availability of microdata has also permitted the use of nonparametric statistical techniques to explore the shape of demand functions. These studies have found that functional forms vary from equation to equation in a demand system, and have provided insights into how improved specifications of parametric models might be formulated. Examples of this type of work are Bierens and Pott-Buter (1990), Nicol (1993a) and Banks, Blundell and Lewbel (1994).

Some other work has highlighted that conditions for perfect aggregation are not met empirically. Such conditions require that demand functions be linear in functions of income and household characteristics. Therefore, models which have demographic effects interacting with income, say, would not be exactly aggregable. There is ample empirical evidence which now demonstrates that these types of interaction are an essential feature of a well-specified demand model. See, for example, Browning and Meghir (1991), Dickens, Fry and Pashardes (1993) and Nicol (1993b).

The foregoing results are significant in themselves, showing as they do that demand model specifications must be much more detailed than those which have been estimated historically. One other result, however, has potentially wider-reaching implications, beyond the applied demand setting. This is the finding that commodity demands are not separable from male and female labour supply variables. This result is quite

general, having been shown for Britain (Browning and Meghir, 1991), Germany (Kaiser, 1993) and Canada (Nicol and Nakamura, 1993). There are implications here for the modelling of labour supply and wage equations which have yet to be explored.

The results discussed above often appear in independent pieces of work. That is, some research focuses on generalising model specification, others on separability issues and others still on modelling demographics. It is clear, however, that all of these influences are important, and must be dealt with simultaneously. Correcting for one model specification error when there are others present does not necessarily provide the researcher with superior estimates. Consider, for example, the case of a model containing errors in variables and autocorrelated errors. Correcting for only one of these specification errors and leaving the other can induce larger inconsistencies than if no adjustments are made for either specification error (Grether and Maddala, 1973, and more recently, Dagenais, 1994).

The purpose of this paper is to conduct an exploratory analysis into a number of model specification errors, with a view to determining the effect each has on a variety of empirical outcomes. In the initial stage, a "trial" data sets are used, to learn about the preferred model structure. These data sets are used to estimate a number of parameterisations of a model, where various specification issues are addressed. In particular, perfect aggregation, separability of commodity demands from labour supply, separability of commodity demands from other goods excluded from the system and household characteristics effects are all modelled. The possible endogeneity of some explanatory variables is controlled for as well. The relative predictive power of models is used as a criterion to assess alternative model performance. This is not the only possible criterion which could be used. Alternative estimates of equivalence scales, elasticities and measures of inequality could all be analysed. These additional issues are the subject of ongoing research.

It is found that exact aggregation and separability of demands from labour and other goods are rejected. Also, household characteristics effects are additional important determinants of demand. These results hold whether estimation is by generalised method of moments (GMM), to control for possible endogeneity of some of the explanatory variables and heteroskedastic disturbances, or by maximum likelihood (ML). One especially important result is that homogeneity and symmetry restrictions are not rejected using either estimation method.

It is also found that exogeneity of the explanatory variables is rejected for relatively restrictive models, but not for the most general model estimated. This has important implications for earlier work (for example, Browning and Meghir, 1991), which emphasised the importance of using instrumental variables estimation, especially when labour force variables were introduced into the demand model specification.

The final set of results involves predictive power of the competing models. Predictive power was, in general, superior when estimation was by ML. This was especially the case for less restrictive model

specifications. Thus, while hypothesis tests indicated the use of GMM estimation was necessary, this was misleading if one's aim was to obtain precise predictions. The relative performance of ML and GMM estimation should therefore be explored in the context of other criteria. In particular, estimation of equivalence scales, elasiticities and other policy-oriented estimates which make use of demand parameter estimates could be sensitive in the same way as observed here for predictive power. This is the subject of ongoing research.

The remainder of the paper is structured as follows. In Section 2, the means of introducing model generality is discussed. Some relevant empirical literature is also discussed, which gives direction to the initial model specification. The data used is briefly discussed in Section 3. Section 4 provides details of the estimated models, hypothesis tests conducted, and gives comparisons of the predictive performance of the various model parameterisations. Section 5 summarises and concludes with some discussion of related research.

2 Model Specification

To deal with all the considerations which have been found to be empirically important could necessitate specification of an extremely complicated model. This complexity can be reduced by making use of a conditional cost function. Such an approach was followed by Browning and Meghir (1991), where they introduced labour supply variables as "conditioning goods". As a consequence of this formulation, it is not necessary to specify the labour supply aspect in any detail. Labour supply variables merely enter the demand functions as additional variables, whose legitimate exclusion can be justified if commodity demands are weakly separable from the labour supply decision. Other goods can be treated in this way if it is felt that the commodities included in the system are not weakly separable from goods which would normally be excluded. This is a convenient way to treat durable goods, for example, where spending on these goods does not reflect actual consumption, but such goods nonetheless influence the demand decision for goods directly included in the system.

Suppose goods (including labour supply) over which consumers make decisions can be partitioned into four types. Goods of direct interest, denoted q, and their prices, p; labour force variables, ℓ and their prices, w; other conditioning goods, g and their prices r; and demographic or household characteristics variables, z. If preferences can be represented by the utility function, $U[q, \ell, g, z]$, the conditional cost function is defined as $c[p, \ell, g, z, u] = \min_q [p \cdot q | U(q, \ell, g, z) = u]$. The properties of these functions are discussed in Pollak (1969) and Browning (1983). It should be noted that q, ℓ, g and z can all be vectors. The conditional, compensated demand functions for q are then just the derivatives of this cost function with respect to p and can be denoted $q_i = f_i[p, \ell, g, z, y]$, where y is total expenditure on the goods comprising the vector q.

The parameterisation of cost function which is used to represent $c[p, \ell, g, z, u]$ is a generalisation of the

price independent generalised logarithmic (PIGLOG) model of Muellbauer (1976),

$$\ln c[p,\ell,g,z,u] = \ln a(p,\ell,g,z) + \frac{b(p,\ell,g,z)}{[f(u) - g(p,\ell,g,z)]} \tag{1}$$

The indirect utility function for this model can be written

$$\ln V[p,y;\ell,g,z] = f^{-1}\{\frac{b(p,\ell,g,z)}{[\ln y - a(p,\ell,g,z)]} + g(p,\ell,g,z)\} \tag{2}$$

This model has been shown to yield rank three, quadratic logarithmic budget-share demand systems with the general form

$$w_i = a_i'(p,\ell,g,z) + \frac{b_i'(p,\ell,g,z)}{b(p,\ell,g,z)}[\ln(y/a[p])] + \frac{g_i'(p,\ell,g,z)}{g(p,\ell,g,z)}[\ln(y/a[p])]^2 \tag{3}$$

by Banks, Blundell and Lewbel (1994). They used a version of this model to estimate a demand system for Britain using FES data. Fry and Pashardes (1992) used a different parameterisation of this model, also with British data, and showed that their variant of the model dominated a number of other popular functional forms which are nested within it, such as the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). The specification in (3) is, however, more general than that in Banks, Blundell and Lewbel (1994), or in Fry and Pashardes (1992). Neither directly included conditioning goods or labour force variables. Browning and Meghir (1991), on the other hand, included the labour force effects, but did not estimate a rank three system, and do not include any additional conditioning goods.

The model in (3) can therefore be seen as encompassing a variety of effects, all of which have been found to be important determinants of demand on their own. The question is whether these effects should enter the model simultaneously, or whether some of the effects are capturing more than one influence. Given a suitably flexible parameterisation for $a(p,\ell,g,z), b(p,\ell,g,z)$ and $g(p,\ell,g,z)$, this issue can be explored. The impact of these separate effects on estimation of by-products of demand estimation itself (such as prediction, elasticities and equivalence scales) can also be explored.

The data to be use in this study are Canadian cross-sectional microdata from several survey years. These data will be described in more detail in the next section. However, it is useful to highlight some properties of these data here, since this has implications for model specification. In particular, the price data to which the expenditure data are matched to estimate the demand system have limited variability within a given survey year. This price variation relates to regional price differentials, and eight survey years are used. This limited variability has implications for the number of parameters which can be identified empirically with respect to price variables and variables which interact with prices. This places some constraints on the specifications of $a(p,\ell,g,z), b(p,\ell,g,z)$ and $g(p,\ell,g,z)$. General functional forms for these are as follows

$$\ln a(p,\ell,g,z) = \alpha_0(z) + \sum_i \alpha_i(\ell,g,z)\ln p_i + \frac{1}{2}\sum_i\sum_j \gamma_{ij}\ln p_i \ln p_j \tag{4}$$

$$b(p,\ell,g,z) = \beta_0(z)\prod_i p_i^{\beta_i(\ell,g,z)} \tag{5}$$

$$g(p,\ell,g,z) = b(p,\ell,g,z)\cdot\lambda(p,\ell,g,z) \tag{6}$$

$$\lambda(p,\ell,g,z) = \lambda_0(z) + \sum_i \lambda_i(\ell,g,z)\ln p_i \tag{7}$$

where adding up requires that $\sum_i \alpha_i(\ell,g,z) = 1, \sum_i \gamma_{ij} = 0, \sum_i \beta_i(\ell,g,z) = 0$; homogeneity requires that $\sum_j \gamma_{ij} = 0$; and symmetry of substitution effects that $\gamma_{ij} = \gamma_{ji}, \forall i \neq j$. Functional forms are required for $\alpha_0(z), \alpha_i(\ell,g,z), \beta_0(z), \beta_i(\ell,g,z), \lambda_0(z)$ and $\lambda_i(\ell,g,z)$. For tractability given the nature of the price data, it was decided to confine the influences of ℓ, g and z to functions, (4)–(6). Several specifications are possible, to allow for the capability to test for: exact aggregation; separability of commodity demands from labour supply; separability of commodity demands from other goods; and the importance of household characteristics effects. In particular, to accommodate specification of a rank three system, (7) is set equal to[2]

$$\lambda(p,\ell,g,z) = \lambda_0 + \sum_i \lambda_i \ln p_i \tag{8}$$

where $\sum_i \lambda_i = 0$ to satisfy adding up. In addition, to introduce the influences of ℓ, g and z into (4)–(6), the following specifications are employed for $\ln a(p,\ell,g,z)$ and $\ln b(p,\ell,g,z)$:

$$\ln a(p,\ell,g,z) = \alpha_0 + \sum_i\sum_k [\alpha_{ik}v_k]\ln p_i + \frac{1}{2}\sum_i\sum_j \gamma_{ij}\ln p_i \ln p_j \tag{9}$$

$$\ln b(p,\ell,g,z) = \beta_0 + \sum_i\sum_k [\beta_{i0} + \beta_{ik}v_k]\ln p_i \tag{10}$$

where the vector $v = [v_1,\ldots,v_K]^T$ is used to represent ℓ, g and z, for notational convenience. From (9) and (10), it can be seen that the influences of ℓ, g and z are confined to the functions $\alpha_i(\ell,g,z)$ and $\beta_i(\ell,g,z)$. Given the above parameterisations for (4)–(7), the following budget-share system can be obtained:

$$\begin{aligned} w_i \;=\; & \sum_k \alpha_{ik}v_k + \sum_j \gamma_{ij}\ln p_j + [\beta_{i0} + \sum_k \beta_{ik}v_k][\ln(y/a[p,v])] + \\ & \{\lambda_i + [\beta_{i0} + \sum_k \beta_{ik}v_k][\lambda_0 + \sum_i \lambda_i \ln p_i]\}[\ln(y/a[p,v])]^2 + \epsilon_i \end{aligned} \tag{11}$$

The random term, ϵ_i, denotes a stochastic disturbance such that $[\epsilon_1,\ldots,\epsilon_n]^T \sim \mathrm{N}(0,\Omega)$. As usual, the covariance matrix of the disturbances is singular, so only $n-1$ equations of the system need be estimated, the parameters of the n'th being recovered via the adding-up conditions. Empirical considerations relating to this stochastic specification will be discussed in detail in Section 4. It should also be noted that the

[2] There is quite a lot of evidence against the hypothesis that $\lambda(p,\ell,g,z) = \lambda_0$. This includes the work on nonparametric demand equation estimation referred to earlier, which shows that the same polynomial in $\ln y$ does *not* enter each equation of a demand system.

interaction of household characteristics variables (in the v vector) with $\ln(y/a[p,v])$ yields a non-exactly aggregable demand system.

A variety of hypotheses can be tested using (11), depending on the restrictions imposed on the parameters of $\alpha_i(\ell,g,z), \beta_i(\ell,g,z)$ and $\lambda(p,\ell,g,z)$. For example, restricting the model to a rank two system requires that $\lambda_0 = \lambda_i = 0$, for all i. Separability of commodity demands from labour supply requires that the parameters in $\alpha_i(\ell,g,z)$ and $\beta_i(\ell,g,z)$ on labour force variables be zero, and so on. A total of six model parameterisations are estimated, and a variety of hypothesis tests carried out. The details of these will be discussed in Section 4.

3 Data

The expenditure data for this study are drawn from the 1969, 1974, 1978, 1982, 1984, 1986, 1990 and 1992 Canadian Family Expenditure Survey public-use microdata files. The research in this paper is the prelude to extensive demand analysis using these data sets, taking into account the considerations focused on in this preliminary study. That is: the need to model non-exactly aggregable demands; control for labour supply effects; conditioning on other non-separable goods' expenditures; and the inclusion of household characteristics effects. Consequently, only a subset of the available data extracted from the surveys are used. Furthermore, it seems clear from other research that it is important to construct data sets with households which are as homogeneous as possible, in terms of their characteristics (Barnes and Gillingham, 1984; Nicol, 1989).

Given the above considerations, twelve household types were extracted from the eight surveys. These types were classified by four family sizes: married couples without children; married couples with one child; with two children; and with more than two children. Also, three types of housing tenure were used to further classify households: renter households; home-owners with mortgages; and home owners without mortgages. For all twelve household types, only those with age of head 18–65 and no self-employed members were included in the samples.

Given all households available of the twelve types, only twenty-five per cent were used in this preliminary study. The intention is to use what is learned here in a follow-up study involving the next fifty per cent of the observations, then use the last twenty-five per cent for an assessment of out-of-sample predictive performance. For brevity in the current paper, results from using three of the twenty-five per cent initial data sets are given. These relate to married couple home-owner-with-mortgage households with no, one and two children (MOR0, MOR1 and MOR2 respectively). The sample sizes were 574, 516 and 951 for MOR1–MOR3 respectively.

The choice of expenditure categories to include in the demand system depends on two important considerations. Large nonlinear systems are difficult to estimate, so the less categories included, the

better. However, the danger in this lies in excluding non-separable goods from the system. This is not a problem in the present study, however, since "other goods" effects are to be captured by the introduction of conditioning goods. Large systems can, of course, be made smaller by aggregating goods. However, inappropriate aggregation of expenditures can lead to misleading inferences (Nicol, 1991, provides some evidence on this in a homogeneity and symmetry testing context). The implications of inappropriate aggregation in estimating models of the kind proposed here is the subject of related, ongoing research.

Given the above considerations, the expenditure categories included in the direct demand system, for which there are actual budget-shares on the left hand side of the estimating equations, are food, alcoholic beverages and clothing. All other expenditures are dealt with as an aggregate conditioning good. Labour force participation status of the male and female household members are included as explanatory (dummy) variables. These labour force participation variables interact with other variables on the right hand side of the estimating equations.

Since the households included in the data sets are already fairly homogeneous, there is limited scope for additional household characteristics effects. The nature of the price data (to be discussed below) reflects regional price differences, so there is not much to be gained by inclusion of regional effect variables. These would be highly collinear with the price data, and there would be great difficulty in empirically identifying all the parameters included. One variable which is likely to capture a significant amount of information about the households' characteristics is the age of the members. Also, variables which have been found to be important in other research: are immigrant status of the head of household; tobacco consumption by the household; and vehicle ownership. These last three effects are introduced as dummy variables.

The price data are taken from the Statistics Canada publication *Consumer Prices and Price Indexes*, Catalogue No. 62-010. These data reflect regional differences in prices in Canada for a variety of goods at a point in time. The prices are rebased to 1978, and normalised to unity at their mean, as is usual in estimating such flexible forms. Further details of the price data, some of which had to be aggregated from less regionally aggregated data, are available on request.

4 Estimation and Results

4.1 Exogeneity of Explanatory Variables

It is becoming increasingly common in applied demand studies using microdata for estimation to be conducted using an instrumental variables approach, rather than ML estimation. This is because of concern over purchase infrequency in some bodies of data, which calls into question the exogeneity of total expenditures (y). In addition, if labour force variables are to be included, there is a strong case for controlling for the possible endogeneity of these variables. Also, in the current study, the conditioning of demands on: expenditures outside the demand system; tobacco consumption; and vehicle ownership calls

for an instrumental variables approach, since such decisions are not independent of allocating expenditures to goods in the food, alcohol, clothing demand sub-system.

Whether instrumental variables estimation is essential in all circumstances of this type, however, should be closely examined. The additional bias introduced in small samples and the increased variability of this estimator are high prices to be paid, if the situation does not require it. In particular, although ML estimation might produce inconsistent estimates if endogeneity of some explanatory variables is not controlled for, these inconsistencies could be made up for by the smaller variances of the ML estimates. Furthermore, estimates based on these parameter estimates such as equivalence scales, elasticities and predictions might not be too inaccurate for the purposes to which they are to be put, whereas instrumental variables estimation based estimates could be much more imprecise.

A first element in assessing the need to employ GMM estimation is to test whether ML estimated parameters, $\hat{\theta}$, are consistent relative to GMM estimates, $\tilde{\theta}$[3]. This is implemented here for each of six model specifications estimated. These model specifications are: a rank three demand system, with all variables in v excluded; and the same model with age of head; labour force variables; other expenditures and immigrant status; tobacco consumption; and vehicle ownership included[4].

To test whether $\hat{\theta}$ is consistent relative to $\tilde{\theta}$, what Davidson and MacKinnon (1993, p.237) refer to as Durbin-Wu-Hausman (DWH) statistics are used. If the respective estimated covariance matrices of $\hat{\theta}$ and $\tilde{\theta}$ are $\hat{\mathrm{V}}(\hat{\theta})$ and $\tilde{\mathrm{V}}(\tilde{\theta})$, then the statistic $[\tilde{\theta}-\hat{\theta}]^T\{\tilde{\mathrm{V}}(\tilde{\theta})-\hat{\mathrm{V}}(\hat{\theta})\}^{-1}[\tilde{\theta}-\hat{\theta}] \overset{A}{\sim} \chi^2(q)$, where q is the rank of the inverse of the covariance matrix difference, under the null hypothesis that the ML estimator is consistent (Hausman, 1978). Unfortunately, in the present application, this statistic does not prove useful, since the matrix difference forming the inner-product is not positive definite[5].

To circumvent the above problem, and still test for the consistency of $\hat{\theta}$, an alternative test statistic suggested by Davidson and MacKinnon (1993, p. 238–242) is used, on an equation-by-equation basis. This test statistic is based on the artificial regressions

$$w_i - f_i(\hat{\theta}) = X(\hat{\theta})b + M_W X(\hat{\theta})^* c + \text{residuals} \tag{12}$$

where $f_i(\hat{\theta})$ is the nonlinear function in the budget-share equations evaluated at the ML estimate, $X(\hat{\theta})$ is a matrix of derivatives of $f_i(\theta)$ evaluated at the ML estimate, and $X(\hat{\theta})^*$ is a sub-matrix of $X(\hat{\theta})$, containing the columns of $X(\hat{\beta})$ which are not in the span of W asymptotically. The matrix W is the matrix of instrumental variables, and $M_W = [I - P_W] = [I - W(W^TW)^{-1}W^T]$. The instruments used are described

[3]Robustness to heteroskedasticity of unknown form of the stochastic disturbances is also allowed for in the implementation of GMM estimation.

[4]The results of testing these competing models is discussed in the next sub-section.

[5]This is a common problem with this test statistic in finite samples.

in Appendix A. Some descriptive statistics on all the variables used are also contained in this Appendix.

A test of the null that c in (12) is zero is an asymptotically valid F test of the null hypothesis that $\hat{\theta}$ is consistent. This statistic has $\{k^*, [N - k - k^*]\}$ degrees of freedom, where k is the row dimension of θ, N is the sample size, and k^* is the number of explanatory variables in the model which are not exogenous.

The results of the tests of consistency of $\hat{\theta}$, or more loosely speaking, exogeneity of X, are presented in Table 1 for the two equations estimated, given that one equation is dropped because of the adding-up conditions. These tests indicate that the null hypothesis cannot be rejected at a significance level of 0.001. However, at a significance level of 0.005, several rejections would be observed. With the relatively large sample sizes seen here, the significance level of 0.001 is perhaps justified, although some researchers would rather be conservative in the other direction. This being the case, estimation of the six models was also conducted using GMM, as mentioned previously. This makes it possible to assess the hypothesis test outcomes for model specification, to be discussed in the next sub-section, on the basis of the competing estimation method. It is also possible, later in Sub-section 4.3, to compare the relative predictive performance of the two estimation methods, for the various model specifications too.

4.2 Model Specification Tests

The model specification tests involved excluding elements of v, and setting $\lambda = [\lambda_0, \lambda_1, \ldots, \lambda_n]^T = 0$, to test the rank three condition[6]. Results of these tests based on ML and GMM estimation are given in Table 2. These indicate that age of head and labour force participation are statistically important determinants of demand, although actual rejections observed differ depending on the estimation method. The null hypotheses that these variables can be excluded are rejected at a significance level of 0.001. Of less importance are the other expenditures, immigrant status, tobacco consumption and vehicle ownership variables. In these cases, rejections are only observed when estimation is by ML. These hypothesis test outcomes could have been influenced by non-exogeneity of the explanatory variables which GMM estimation has controlled for. Consequently, it is only possible to state conclusively taht age of head and labour force variables are important. Also, the fact that these variables interact with $\ln(y/a[p, z])$, and that their coefficients are non-zero is evidence against exact aggregation, as has been found in earlier research.

One final point before leaving this sub-section concerns the rank of the demand system required. Lewbel (1989) found that budget-shares linear in $\ln(y/a[p, z])$ (rank two demand systems) were adequate to fit United Kingdom FES data, when the households in the samples were fairly homogeneous in terms of their characteristics. Here, households in a given sample are extremely homogeneous, and GMM estimation

[6]The validity of homogeneity and symmetry restrictions was assessed for each of the six models estimated, and found to be supported by these data. In an earlier study using renters only data (Nicol, 1995), these restrictions were not rejected either. Since such tests can be influenced greatly by model specification error, these non-rejections are encouraging for the model specifications being employed here.

indicates that a rank two specification is adequate. ML estimation, on the other hand, suggests a rank three model specification. This latter result is, again, probably influenced by the exogeneity status of the explanatory variables across estimation methods. That is, given the outcome of the tests in Table 1, the test results in Table 2 based on GMM estimation should carry greater weight. Being able to exclude these quadratric terms would greatly simplify estimation, and if samples are composed of fairly homogeneous household groups, this seems to be justified.

4.3 Predictive Power Under Alternative Estimation Conditions

Even in situations where, say, exogeneity of explanatory variables is rejected, and so one must employ GMM in estimation to obtain consistent parameter estimates, it is still possible that useful estimates could be obtained using the (inconsistent) ML estimates, in certain respects. This possibility is explored in this sub-section, in the context of the predictive power of the various models estimated, when these two alternative estimation methodologies are used.

To explore predictive power, goodness-of-fit pseudo χ^2 statistics are calculated. The method is to take the fitted values of budget-shares under the alternative estimation regimes, and compare the performance of the predictions for different household sub-groups. The sub-groups focused on are labour force participation based. Since each household is comprised of a male and female, there are four strata of labour force participation in any given household. Both working outside the home; one working outside the home; and both not working outside the home.

Denote ML predictions by $\hat{w}_i^{ml}$ and GMM predictions by $\hat{w}_i^{iv}$, and actual shares by w_i^A. Then the deviations of actual from predicted shares are d_i^{ml} and d_i^{iv} for ML and GMM respectively. The sampling distributions of the means of these deviation vectors are then normally distributed, by the Central Limit Theorem. There are three shares, but only two independent pieces of information. These sample means can be converted to standard normals, and χ^2 statistics calculated. That is, we have $\sum_i^3 (\bar{z}_i^{ml})^2 \sim \chi^2(2)$ and $\sum_i^3 (\bar{z}_i^{iv})^2 \sim \chi^2(2)$ also. Here, $\bar{z}_i^{ml} = [1/N] \sum_h^N \sqrt{N} d_i^{ml} / \hat{\sigma}_{ml}$, where N is the number of households in a given labour force participation stratum, h indexes households and $\hat{\sigma}_{ml}$ is the sample standard deviation of the vector of deviations, d_i^{ml}. Analogous definitions can be made for the corresponding GMM case.

The above χ^2 statistics were calculated for each of the six models and estimation régime, across all four labour force participation strata. These statistics are an indication of the goodness-of-fit of the respective estimates, and are presented in Table 3. From this, it can be seen that predictive power is superior almost always when estimation is by ML. This confirms the earlier discussion that the GMM estimates could possibly have larger variability than the ML estimates, even though the latter could exhibit inconsistency.

Only for the MOR0 data set is performance relatively equal for ML and GMM prediction. For MOR1 and MOR2, GMM predictions are superior in the majority of comparisons. One of the reasons for using

GMM was the possible endogeneity of labour force participation variables. One might therefore expect that at least for models with labour force participation variables present, GMM prediction might perform fairly well. This would be for Models 3–6. It can be seen, however, that the NL3LS predictions are poorer than the ML predictions in almost all of these cases.

These results indicate that there are some dimensions at least where it matters a great deal whether estimation is by ML or by GMM. It will be interesting to explore whether this kind of result holds for other estimates implied by demand parameters, such as equivalence scales, elasticities and measures of inequality. These are all uses to which estimated demand parameters are put. Research on these other issues is continuing.

5 Summary and Conclusions

In this paper, a general, rank three demand system is estimated. This model also controls for: the possibility of non-exact aggregation; non-separability of labour force participation effects; non-separability of other goods, not included directly in the demand system; and the influence of household characteristics effects. Recent research has focused on the possible need to control for non-exogeneity of certain explanatory variables. This issue is analysed by conducting tests for the consistency of ML estimates. These tests indicate that non-exogeneity cannot be ruled out. As a result, comparative results are computed of various model specification hypothesis tests using both ML and GMM estimated parameters.

It is found that most of the effects modelled are important determinants of demand, with the possible exception of the rank three requirement. This is true whether the estimation methodology used is ML or GMM. Since the households in the data sets used are fairly homogeneous, this result supports a similar finding by Lewbel (1989) using British data. In summary, it thus does not seem to matter for general model specification testing purposes which estimation method is used. However, perhaps this does matter along some other dimensions of interest to applied demand researchers.

A comparison of predictive power under the alternative estimation régimes of ML and GMM was then conducted, and this indicated that the former method produces superior predictions. Thus, while there seems to be evidence that GMM estimation is required on the basis of non-exogeneity of some explanatory variables, this does not seem to make a difference for the purposes of hypothesis testing, or for prediction. Whether this result generalises to other estimates such as equivalence scales and elasticities remains to be analysed.

The work in this paper is of a preliminary nature, and is designed to provide some insights into the direction of future research. The data set used here was deliberately chosen to ensure that a detailed analysis could be performed as to the importance of certain model specification issues. At the same time, sufficient additional data remains to analyse a more refined model indicated by the present study. Such a

model need not be a rank three system if households groups are homogeneous, but should be non-exactly aggregable, control for labour force participation of adult household members and model some additional household characteristics effects. This research is continuing.

Some additional issues, which have not been dealt with here, but which are also likely to be of some importance is the method of inclusion of the conditioning good on all other expenditures. It is likely that the effects associated with a dollar spent on durable goods is not the same as a dollar spent on nondurable goods not included directly in the system. Therefore, there is a need to explore whether an aggregate conditioning good is an adequate specification. Finally, although four important and distinct types of effects are modelled, this is done in a fairly specific way. The function $\lambda(p)$ could be modelled in different ways. However, it will only ever be possible to generalise this and the other functions to a fairly limited extent given the shortcomings of the price data available. On the other hand, the Consumer Expenditure Surveys (CES) for the United States have a much more rich set of possibilities in terms of the complexity of price effects which can be modelled. These data are being used to explore some further issues in related research.

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Table 1: Durbin-Wu-Hausman Test Statistics for the Consistency of the MLE

(MOR0) Model	Food Equation Test Statistic	Degrees of Freedom	Cumulative Density	Alcohol Equation Test Statistic	Degrees of Freedom	Cumulative Density
1	5.542	4,560	0.999	1.130	4,560	0.659
2	2.250	5,556	0.952	0.831	5,556	0.472
3	3.004	10,548	0.999	0.722	10,548	0.296
4	2.230	14,538	0.995	1.391	14,538	0.848
5	1.413	18,528	0.881	1.095	18,528	0.647
6	1.365	21,522	0.871	0.834	21,522	0.322

(MOR1) Model	Food Equation Test Statistic	Degrees of Freedom	Cumulative Density	Alcohol Equation Test Statistic	Degrees of Freedom	Cumulative Density
1	2.816	4,502	0.975	0.241	4,502	0.085
2	4.520	5,498	0.999	0.258	5,598	0.064
3	1.059	10,490	0.607	0.927	10,490	0.492
4	2.030	14,480	0.986	0.850	14,480	0.385
5	2.016	18,470	0.992	0.948	18,470	0.480
6	1.418	21,464	0.896	0.660	21,464	0.127

(MOR2) Model	Food Equation Test Statistic	Degrees of Freedom	Cumulative Density	Alcohol Equation Test Statistic	Degrees of Freedom	Cumulative Density
1	1.488	4,937	0.796	1.480	4,937	0.794
2	1.489	5,933	0.809	0.654	5,933	0.342
3	1.746	10,925	0.934	2.943	10,925	0.999
4	1.618	14,915	0.932	1.330	14,915	0.818
5	2.469	18,905	0.999	1.751	18,905	0.973
6	1.946	21,899	0.993	1.893	21,899	0.991

Note: The test statistics are asymptotically distributed as F, with the stated degrees of freedom. The method of calculating these statistics is explained in detail in the text.

Table 2: Model Specification Tests Based on ML and GMM Estimation

Tests Based on ML Estimation.

	Excluding Q		Excluding A		Excluding L	
Data	Test Statistic	Prob. Value	Test Statistic	Prob. Value	Test Statistic	Prob. Value
MOR0	26.396	0.263E-04	35.806	0.317E-06	24.777	0.558E-04
MOR1	19.436	0.645E-03	18.512	0.980E-03	21.848	0.215E-03
MOR2	3.290	0.511E+00	16.888	0.203E-02	17.368	0.164E-02

	Excluding C		Excluding V		Excluding T	
Data	Test Statistic	Prob. Value	Test Statistic	Prob. Value	Test Statistic	Prob. Value
MOR0	17.450	0.158E-02	3.932	0.415E+00	29.512	0.615E-05
MOR1	22.500	0.159E-03	1.482	0.830E+00	25.272	0.444E-04
MOR2	10.710	0.300E-01	8.532	0.739E-01	8.438	0.768E-01

	Excluding I	
Data	Test Statistic	Prob. Value
MOR0	5.397	0.249E+00
MOR1	11.392	0.225E-01
MOR2	8.326	0.803E-01

Tests Based on GMM Estimation.

	Excluding Q		Excluding A		Excluding L	
Data	Test Statistic	Prob. Value	Test Statistic	Prob. Value	Test Statistic	Prob. Value
MOR0	0.333	0.954E+00	20.245	0.447E-03	16.824	0.320E-01
MOR1	4.881	0.181E+00	16.724	0.219E-02	31.919	0.963E-04
MOR2	1.200	0.753E+00	13.641	0.853E-02	31.376	0.120E-03

	Excluding C		Excluding V		Excluding T	
Data	Test Statistic	Prob. Value	Test Statistic	Prob. Value	Test Statistic	Prob. Value
MOR0	9.813	0.437E-01	3.175	0.529E+00	1.388	0.846E+00
MOR1	11.180	0.246E-01	0.996	0.910E+00	11.376	0.226E-01
MOR2	5.206	0.267E+00	4.798	0.309E+00	1.285	0.864E+00

	Excluding I	
Data	Test Statistic	Prob. Value
MOR0	0.499	0.974E+00
MOR1	12.758	0.125E-01
MOR2	1.520	0.823E+00

Notes to Table 2

1. In the headings to the test statistics, "Excluding Q" and so on indicates the variables being excluded are: Q, quadratic terms in total expenditures; A, age of head; L, labour force variables dummies; C, other expenditures conditioning good; V, vehicle ownership dummy; T, tobacco consumption dummy; and I immigrant status dummy.

2. The degrees of freedom for the tests are 8 for L; 4 for A, C, V, T and I; and 3 for Q.

3. The test statistics based on ML estimation are likelihood-ratio test statistics; those based on GMM estimation are Wald statistics. All statistics are distributed as χ^2, with the degrees of freedom indicated.

Table 3: Comparisons of Goodness-of-Fit of ML and GMM Estimated Models Using χ^2 Statistics. MOR0 Data Set.

Labour Force Status	Model 1, ML	Model 1, GMM	Model 2, ML	Model 2, GMM
(1,1)	29.777	17.823	5.533	2.083
(1,0)	50.530	45.227	14.684	12.656
(0,1)	10.371	1.140	0.151	0.841
(0,0)	4.831	0.640	1.054	4.447

Labour Force Status	Model 3, ML	Model 3, GMM	Model 4, ML	Model 4, GMM
(1,1)	0.317	1.669	0.126	1.754
(1,0)	0.895	0.829	0.268	0.250
(0,1)	6.175	12.367	1.949	5.865
(0,0)	5.719	6.717	1.648	0.055

Labour Force Status	Model 5, ML	Model 5, GMM	Model 6, ML	Model 6, GMM
(1,1)	0.122	1.596	0.126	0.993
(1,0)	0.273	0.142	0.209	0.648
(0,1)	1.999	4.323	1.655	2.444
(0,0)	1.590	0.106	1.328	0.398

Table 3: *continued*, MOR1 Data Set.

Labour Force Status	Model 1, ML	Model 1, GMM	Model 2, ML	Model 2, GMM
(1,1)	10.658	16.230	7.996	15.258
(1,0)	23.111	18.216	19.190	8.087
(0,1)	0.350	0.428	0.347	0.215
(0,0)	42.084	3.601	17.698	4.670

Labour Force Status	Model 3, ML	Model 3, GMM	Model 4, ML	Model 4, GMM
(1,1)	0.009	6.160	0.005	6.134
(1,0)	0.028	1.213	0.014	0.224
(0,1)	1.099	2.062	0.267	2.149
(0,0)	5.691	0.486	2.880	10.575

Labour Force Status	Model 5, ML	Model 5, GMM	Model 6, ML	Model 6, GMM
(1,1)	0.004	7.138	0.006	4.740
(1,0)	0.011	0.022	0.012	0.243
(0,1)	0.241	0.347	0.558	0.251
(0,0)	6.412	4.370	1.896	0.608

Table 3: *concluded*, MOR2 Data Set.

Labour Force Status	Model 1, ML	Model 1, GMM	Model 2, ML	Model 2, GMM
(1,1)	11.606	15.830	10.861	15.046
(1,0)	17.169	8.082	16.487	11.354
(0,1)	0.093	0.237	0.044	0.213
(0,0)	1.119	2.268	0.452	2.019

Labour Force Status	Model 3, ML	Model 3, GMM	Model 4, ML	Model 4, GMM
(1,1)	0.018	0.553	0.009	1.257
(1,0)	0.024	1.354	0.007	0.123
(0,1)	0.875	3.828	0.349	0.198
(0,0)	1.355	0.042	0.449	0.740

Labour Force Status	Model 5, ML	Model 5, GMM	Model 6, ML	Model 6, GMM
(1,1)	0.008	1.151	0.006	0.399
(1,0)	0.010	3.190	0.007	0.165
(0,1)	0.308	1.521	0.219	27.624
(0,0)	0.410	0.712	0.293	7.714

Notes:

1. The four labour force status classes, (1,1), (1,0), (0,1) and (0,0) refer to the values taken by the labour force participation dummy variables. The cell (1,1) indicates labour force participation outside the home by both the male and female member. The cell (1,0) indicates labour force participation outside the home by the male and non-participation by the female member, and so on.

2. Sample sizes in the respective cells in the order listed are: 391, 140, 21 and 22 for MOR0; 357, 150, 4 and 5 for MOR1; and 530, 403, 10 and 8 for MOR2.

3. The statistics are disrtibuted as $\chi^2(2)$. The critical value of a $\chi^2(2)$ statistic at a 0.001 significance level is 13.816; and at a 0.01 significance level, it is 9.210.

Appendix A
Instrumental Variables and Descriptive Statistics

The instruments for GMM estimation were: ages of the male and female adult household members, and these variables squared; five regional dummy variables for the Atlantic Provinces, Québec, Ontario; the Prairie Provinces; and British Columbia; prices of food, alcoholic beverages and clothing; squares and cross-products of the price variables; eight occupational dummy variables for male and female adult household members; five educational status dummy variables for male and female household members; a time trend; income after tax and its square; income after tax interacted with all adult age variables; personal taxes; and government transfer payments. Descriptive statistics on the variables are as follows:

Table A1: Descriptive Statistics for Non-Dummy Variables

	MOR0		MOR1		MOR2	
Variable	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Age of Male	42.8	13.3	35.9	8.9	36.6	6.4
Age of Female	40.7	13.5	33.6	8.5	34.1	6.1
After-tax Income	$25420	$20676	$24311	$19029	$23067	$17999
Personal Taxes	$8693	$8154	$7457	$6833	$7254	$6691
Govt Transfers	$2183	$2139	$1958	$1733	$1965	$1849

Table A2: Count Summary for Dummy Variables

	MOR0		MOR1		MOR2	
Variable	Male	Female	Male	Female	Male	Female
OM1, OF1	77	38	75	23	125	22
OM2, OF2	116	109	116	108	211	142
OM3, OF3	43	167	31	121	73	210
OM4, OF4	43	27	38	22	73	38
OM5, OF5	36	38	45	46	84	69
OM6, OF6	204	29	200	37	352	60
OM7, OF7	42	150	7	143	17	390
OM8, OF8	13	16	4	16	16	20
EM1, EF1	85	68	50	44	81	67
EM2, EF2	244	274	232	237	435	503
EM3, EF3	66	59	62	68	98	98
EM4, EF4	97	104	74	108	177	188
EM5, EF5	82	69	98	59	160	95

Notes:

1. OM1–OM8 and OF1–OF8 refer to the occupation dummy variables for males and females respectively. These are defined as: managerial/administrative; professional/technical; teaching; clerical; sales; services; farming, fising, forestry, mining, processing, fabrication, construction and all other; and not working.

2. EM1–EM5 and EF1–EF5 refer to the education dummy variables for males and females respectively. These are defined as: less than nine years schooling; some/completed secondary; some post-secondary; post-secondary certificate or diploma; and university degree.

3. The breakdown of regional dummies, REG1–REG5, are: MOR0, 92, 93, 138, 180, 71; MOR1, 98, 129, 126, 110, 53; MOR2, 180, 193, 247, 232, 99.

4. More detailed information on these variables can be obtained from the documentation which comes with the FAMEX public-use tapes.