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## RETIREMENT SAVINGS AND PUBLIC PENSIONS

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## Abstract

An overlapping generations general equilibrium model is presented to quantify the impact of introducing a public pension program on the retirement savings motive of individuals in life-cycle economies. Aggregate savings rates are calculated and compared for model economies for cases of varying levels of benefits. Model economy results suggest that, for modest pensions programs like the Canada Pension Plan, aggregate savings and domestic asset holdings are reduced by twenty percent. For more generous plans, like US Social Security, aggregate savings and domestic asset holdings are reduced by forty percent.

*Keywords:* retirement saving, public pensions, overlapping generations.

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## I. Introduction

This paper presents an overlapping generations general equilibrium model to quantify the impact of introducing a public pay-as-you-go pension program on aggregate savings in life-cycle economies. Aggregate savings rates are calculated and compared for model economies with varying retirement benefit levels. Model results show that the personal retirement savings motive is significantly reduced with the introduction of a very modest pension program. In the model economies investigated, a pension program, with a replacement ratio of 14% of pre-retirement earnings, reduces the amount of savings and domestic asset holdings by more than twenty percent. A more generous program, with a replacement ratio of 28% of pre-retirement earnings, reduces the amount of domestic asset holdings by more than forty percent, and the net domestic savings rate falls by almost a half.

In the major industrialized nations, savings rates have generally declined since the 1970s. In the US, the household savings rate has fallen from eight percent in the 1970s, to nil in 1999 (Gordon (2000:511)).<sup>1</sup> The United States went from being a net supplier of capital to the rest of the world, to a net debtor (Berheim and Shoven, 1991:9). Private savings rates in Japan fell from 19.3% over 1960-73, to 12.1% over 1984-7 (Bosworth, 1991:32). In Canada, personal net savings rates fell from 9.2% in 1975 to 2.3% in 1999 (CANSIM matrices 8614, 8629). During this period, industrialized nations have instituted and/or restructured safety-net programs of various types, including the expansion of unemployment insurance programs, social security and public pension programs.

Research by Gokhale, Kotlikoff and Sabelhaus (1996) has pointed out that the introduction of Social Security in the US has reduced poverty of the elderly considerably. In the early 1960s, the typical seventy year-old was consuming two-thirds the amount of consumption of the typical thirty year-old. Thirty years later, the typical seventy year-old consumed one-fifth more than a thirty year-old. The redistribution of income from the young to the old has resulted in increased rates of consumption and has depressed savings rates. Econometric investigations on the effect of pension systems on savings have not

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<sup>1</sup> Adjustments for stock market gains in the 1980s and early 1990s lead to the conclusion that the gains-inclusive savings rate declined somewhat in the years leading up to the mid-1990s, with a surge during the market boom of the 1990s (Gale and Sabelhaus (1999)). However, in light of recent economic events in 2001, it is unclear what general pattern will emerge for personal savings.

been able to determine conclusively the reduction in savings as a result of the introduction of pay-as-you-go plans. Feldstein (1976) concluded that increases in social security wealth in the US reduced personal savings by forty percent. Subsequent research by other authors, using different econometric specifications, has reduced this figure (for example, Munnell (1977) included the effect of changes in the unemployment rate on savings). Perhaps pensions increase saving by promoting a retirement age and educating the public about the need to save for the future (Cagan (1965), Katona (1964), Leimer and Lesnoy (1982)).

In Canada, the aggregate net savings rate fell from a high of 16.1% in 1974, to a low of 4.4% in 1993. The components of the aggregate savings rate in Canada from 1962-2000 are depicted in Figure 1. To what extent can the introduction of the Canada Pension Plan in 1966 explain the decline in the savings rate? A clear majority of Canadians believe that an essential government activity is the provision of a social safety-net and the implementation of measures to ensure a basic standard of living (Peters, 1995: 105). The Canadian Public Pension program was developed in 1966 to assist the elderly during their retirement years, and to supplement the Old Age Security program, first introduced in 1927. The Guaranteed Income Supplement and the Spouse's Allowance were added in 1967 and 1975 respectively. These programs provide substantial help to the elderly and have helped to reduce the incidence of poverty among the aged from 41% in 1969 to 7% in 1994. The current policy debate focuses on the fact that, for the public pension system, "whatever the initial intentions, the system is largely an underfunded one" (Banting and Boadway, 1996:20). While the underfunded nature of public pensions has policy implications for the future sustainability of the program, it also has implications regarding the incomplete transfer of savings from the private sector, to the government sector. The CPP is financed by program premiums through payroll taxes. Surpluses or deficits of this program are included in the annual operating deficit or surplus of the government and will affect the level of government savings. The public pension program will affect the individual's retirement savings motive, reducing the need to personally save to finance consumption when no longer in the workforce. To what extent has the introduction of public pensions reduced the aggregate savings rate? This work is intended to complement the current policy debate on policy reform, in light of the

massive public expenditures, both current and expected, on social programs, and the debate over the effectiveness of new initiatives designed to increase private saving.

This paper presents an overlapping generations general equilibrium model of life-cycle economies to investigate the impact of a public pay-as-you-go pension program on personal savings in the aggregate. The model economies are populated by agents of overlapping generations. Agents have limited and uncertain lifetimes and face earnings uncertainty. Model results indicate that public pensions programs significantly reduce the private retirement savings motive of individuals. In the model economies investigated, a modest pension program, like the Canada Pension Plan, with a replacement ratio of 14% of pre-retirement earnings, reduces the net personal savings rate and domestic asset holdings by about twenty percent. A more generous program, with a replacement ratio of 28%, more like that of US Social Security, reduces the net personal savings rate and the amount of domestic asset holdings by more than forty percent. As a consequence, in a small open economy, foreign indebtedness increases, and in a closed economy, the equilibrium interest rate increases and wages fall.

## **II. Overlapping Generations Life-Cycle Economies**

The overlapping generations framework has been used to examine social security, saving and capital accumulation, and public pension reform (for example, Hubbard and Judd (1987), Hubbard, Skinner and Zeldes (1995), Bohn (1999), Huggett and Ventura (1999), Emery and Rongve (1999)). It has been found that, in model economies, social security and pensions can have a considerable impact on welfare and distort savings incentives. This framework is rich enough to incorporate uncertain finite lifetimes, heterogeneity in age and employment status, and other changing characteristics over the life cycle.

### ***The Environment***

The modeling environment is similar to that used by Huggett (1996), and Wilson (2001b). The model economies to be examined are populated by overlapping generations of rational, forward-looking agents. Agents are heterogeneous in age and employment history. Agents live to a maximum of  $I$  periods, start work at  $I_w$ , and retire at  $I_r$ . The model takes as given that agents retire at age 65, whether a pension program exists or

not.<sup>2</sup> The population distribution by age cohort at a point in time is described by the vector  $\mathbf{m}$ , where  $\mathbf{m} = [m_1 \ m_2 \ \dots \ m_j]^T$ . The age  $i$  cohort consists of  $m_i$  agents. To discriminate between agents with different employment history, agents within a cohort are indexed by  $h_i$ , where  $h_i \in (1, m_i)$ . Agents face uncertain lifetimes, but know age-dependent survival probabilities. The probability of surviving to from age  $i$  to age  $i+1$  is  $s_i$ . The unconditional probability of surviving to age  $k$  is  $s(k) = \prod_{j=1}^{k-1} s_j$ . In a steady-state, age  $i$  agents will make up a constant fraction of the population at any point in time, and the population growth rate will be  $n$ .

At the aggregate level, there is no uncertainty. Production is deterministic and is governed by a constant returns to scale technology of the following form:

$$Y_t = F(K_t, L_t) = BK_t^a (A_t L_t)^{1-a}. \quad (1)$$

Labour-augmenting technological progress,  $A_t$ , grows at a constant rate  $g$ .  $B$  is a scale parameter. Capital depreciates at a constant rate  $d$ . The model economies to be investigated will be small open economies, where the interest rate will be set exogenously, and will be set to a constant.

Agents maximize expected utility over a standard composite good, with time-separable preferences and discount factor  $\mathbf{b}$ . The composite good can be used as capital, and earns a risk-free one period return per unit,  $R$ . Agents maximize expected lifetime utility, given by:

$$\max E_{I_w} \left( U(c_{I_w}) + \sum_{j=1}^{I-I_w} \mathbf{b}^j s(j+I_w-1) U(c_{j+I_w}) \right), \quad \text{where } U(c) = \frac{c^{1-s}}{1-s}. \quad (2)$$

Agents are endowed with labour efficiency units that are dependent on age, denoted by  $e_i$ . Adults of working age face the probability  $\mathbf{p}_i^e$  of being employed, and  $\mathbf{p}_i^u = 1 - \mathbf{p}_i^e$  of being unemployed in any given period. The probability of being employed in a given period for an individual is independent of being employed in any other time period. If employed,  $l(h_i) = 1$ . If unemployed,  $l(h_i) = 0$ . In these economies, while there is no aggregate uncertainty, there exists uncertainty at the individual level.

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<sup>2</sup> The labour force participation rate of males aged 65 and older fell steadily from 30% in 1960 to 20% in 1970, and has fluctuated around 10% in the last decade. The induced retirement effect of pensions is explored in Feldstein (1974), and will be left to be examined in future work in the modeling framework described herein.

The government provides retirement benefits and employment insurance to agents in the economy. Individuals who survive past the age of  $I_{r-1}$  receive retirement benefits at a rate equal to the benefit rate,  $b_{i,t}$ , times a level  $e_i$  indicative of the employment history of the average individual of that cohort when  $i \geq I_r$ .<sup>3</sup> Unemployed agents receive a benefit equal to a proportion,  $b_{i,t}$ , of their expected earnings if employed. The government finances the employment insurance and the pension programs by levying payroll taxes equal to a proportion,  $p_t$ , on labour earnings.

An agent of age  $i$  chooses consumption,  $c_{i,t}$ , and asset holdings,  $a'_{i,t} = a_{i+1,t+1}$ , in period  $t$ , that are carried into the next period, with the following constraints:

$$c_{i,t} + a'_{i,t} \leq W_t = R_t a_{i,t} + (1 - p_t) w_t l(h_i) e_i + b_{i,t} w_t (1 - l(h_i)) e_i + T_t, \text{ and}$$

$$c_{i,t} \geq 0, \text{ and } a'_{i,t} \geq 0 \text{ for all } i, \text{ and } t. \quad (3)$$

In this context, the value of  $T_t$  is determined by the amount of accidental bequests left by those who die in period  $t$ , before the age of  $I$ . Accidental bequests have been treated several different ways. Rios-Rull (1994) introduces a market that has agents write contracts with members of their own age cohort to share wealth or debts of those members who die before age  $I$ . Storesletten (2000) has accidental bequests donated to newborns in a lump-sum transfer. Huggett (1996) treats accidental bequests as fully taxed and redistributed amongst surviving agents in the economy. The most appropriate modeling strategy would be that of Huggett, as inheritances tend in practice to be distributed amongst family members of varying ages. The treatment here follows Huggett; the value of these bequests is distributed equally amongst the adult population (those of age  $I_w$  or older).

Agents are liquidity constrained and cannot hold negative net assets. Agents are not capable of borrowing without collateral greater than or equal to the amount borrowed, but are allowed to mortgage capital. In this case agents are unable to borrow against the promise of future income to finance consumption, and cannot hold negative net asset levels. In the life-cycle model economy with survival uncertainty, this assumption restricts agents' borrowing so that they do not accumulate debts and die before repayment.

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<sup>3</sup> This is done for simplicity, instead of keeping track of each individual's employment history and having

The government levies a payroll tax equal to a rate  $p_t$  of employment earnings to finance the employment insurance and pension programs. In the steady state, the government budget must be balanced. Total revenues from the payroll tax will be equal to the expenditures in providing employment insurance benefits to the unemployed and pensions to those who are retired. The government budget constraint is:

$$\sum_{i,h_i} (pp_i^e \mathbf{m}_i we_i) - (b_i p_i^u \mathbf{m}_i we_i) = 0.^4 \quad (4)$$

### **Steady-State Equilibrium**

In order to describe a steady-state equilibrium in model economies in which there is both population and productivity growth, the variables must be transformed as follows:

$$\begin{aligned} \tilde{L}_t &= L_t / L_t, \tilde{K}_t = K_t / L_t A_t, \tilde{T}_t = T_t / A_t, \tilde{a}_t = a_t / A_t, \tilde{c}_t = c_t / A_t, \tilde{\mathbf{w}}_t = \mathbf{w}_t / A_t, \\ \tilde{\mathbf{m}}_t &= \mathbf{m}_t / L_t, \tilde{N}_t = N_t / L_t. \end{aligned}$$

In the steady state, the transformed variables are constant over time, and the untransformed variables grow at constant rates.

Definition: A steady-state open economy equilibrium is  $\{ \{ \tilde{c}_{i,h_i}, \tilde{a}'_{i,h_i} \}_{\forall i,h_i},$

$\tilde{\mathbf{w}}, R, \tilde{K}, \tilde{K}_d, \tilde{K}_f, \tilde{L}, \tilde{T} \}$  such that:

- (1)  $\tilde{c}_{i,h_i}, \tilde{a}'_{i,h_i}$  are optimal decision rules  $\forall i, h_i$ ;
- (2) Input markets are competitive:  $\tilde{\mathbf{w}} = F_2(\tilde{K}, \tilde{L})$  and  $R = 1 + F_1(\tilde{K}, \tilde{L}) - \mathbf{d}$ , with  $R$  set on international markets, and here assumed constant;
- (3) The allocations are feasible:
  - (i) Domestic asset holdings equal domestic capital
$$\tilde{K}_d = \sum_{i,h_i} \tilde{\mathbf{m}}_{i,h_i} \tilde{a}_{i,h_i} / (1+n);$$
  - (ii) Foreign capital is the difference between total capital and domestic capital:  $\tilde{K}_f = \tilde{K} - \tilde{K}_d$ ;

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benefits dependent on an individual's employment history.

<sup>4</sup> The model economies in many cases have rates of interest in excess of the rates of population growth, meaning that steady-state deficits will cause a continually rising debt to output ratio.

(iii) Total aggregate wealth equals total consumption and next period domestic asset holdings:

$$\tilde{w}\tilde{L} + \sum_{i,h_i} \tilde{m}_{i,h_i} \tilde{a}_{i,h_i} R / (1+n) = \sum_{i,h_i} \tilde{m}_{i,h_i} \tilde{c}_{i,h_i} + \sum_{i,h_i} (1+g) \tilde{m}_{i,h_i} \tilde{a}'_{i,h_i};$$

(4) The number of employed is  $E = \sum_i \mathbf{p}_i^e \mathbf{m}_i$ , the number of unemployed

is  $U = \sum_i \mathbf{p}_i^u \mathbf{m}_i$ . Effective labour supply,  $\tilde{L} = \sum_i \tilde{m}_i \mathbf{p}_i^e e_i$ ;

(5) Transfers to the adult population equal accidental bequests:

$\tilde{T} = \sum_{i,h_i} \tilde{m}_{i,h_i} (1-s_i) R \tilde{a}_{i,h_i} / \tilde{N}_a (1+n)$ , where  $N_a$  denotes the number of adults in the steady-state population.

(6) The government budget is balanced - the total amount of premiums received is equal to total benefits paid,  $\sum_i (p \mathbf{p}_i^e - b_i \mathbf{p}_i^u) \tilde{m}_i w e_i = 0$ .

The method of computing the steady state is similar to the method described in Wilson (2001b). First, the expected total labour supply in the economy is calculated. Then given benefit rates,  $b_i$ , the expected total amount of pension and employment insurance benefits is calculated. The premium rate,  $p$ , is then set and a guess of the transfer,  $T$ , is made. The level of optimal asset holdings of agents in the economy can be computed recursively, given the level of transfers. Then, the values of accidental bequests (transfers) are calculated and compared to the guessed level of transfers. If the two values are equal the procedure is completed, otherwise, the guessed level of transfers is updated and the computational procedure is repeated until convergence is achieved.<sup>5</sup>

### III. Model Calibration

In the model economies investigated, each period corresponds to one year. Agents have uncertain lifespan but commonly know age-specific survival rates. The maximum lifespan is set at 95 years, and all agents die before their 95<sup>th</sup> birthday. Agents begin supplying labour and making decisions regarding consumption and asset holdings at 20 years of age. Agents fully withdraw from the labour force at 65. There is no aggregate uncertainty, but there exists employment uncertainty at the individual level. Agents save

to supplement income received from the government employment insurance and pension programs, to smooth consumption over the life cycle.

### **Population**

In the model economies examined, a no-migration scenario was assumed for simplicity.<sup>6</sup> Survival rates were derived from the ratio of the population to the population plus deaths at each age. This ratio was calculated using the 1987 statistics for Canada, from the CANSIM database (Statistics Canada). Age-specific rates can be calculated for single ages from birth to 69 years of age. Age-specific survival rates for agents in the cohort groups, 70-74, 75-79, 80-84, 85-89 and 90 and older, were assumed to be constant for each age over the age category. The survival rate of 94 year-olds is set to 0. Fertility rates were constructed using the 1987 age-specific fertility rates of females from Statistics Canada (1993) in five-year cohorts from 15-19 years of age, to 45-49 years of age. Fertility rates were assumed constant over all ages in each cohort group. Over 1960-2000, the sex ratio of births in Canada averaged 1.055 males for every female. For the model economies, the male-female split among newborns is set to 1.055:1.

For simplicity, the steady-state population distribution for males was then used for the model economy. This was done to avoid modeling female fertility decisions, and labour force participation decisions during child-rearing years. Given the Canadian population parameters, and a no-migration process, the population growth rate in the steady-state was calculated to be  $-0.79\%$ . Another model economy was calculated using the higher fertility rates of Canadian females in 1970, which results in a steady-state population growth rate of  $0.43\%$ .<sup>7</sup> These population distributions are compared with the distributions of the Canadian population in 1987 and 2000 in Figure 2.

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<sup>5</sup> A more detailed description is included in the appendix.

<sup>6</sup> This is done to eliminate complications that arise in trying to model differences in immigrant wealth holdings upon arrival and labour market characteristics in economies with employment uncertainty. See Storesletten (2000) for an illustration of modeling these differences in economies without employment uncertainty.

<sup>7</sup> The steady-state population calculation procedure is described in the appendix.

### ***Earnings and Employment Uncertainty***

The age-specific labour endowment profile is constructed using the average earnings levels of CPP Male contributors in 1987, from Health and Welfare Canada (1989:Table 13), for agents between the ages of 20 and 64 (inclusive). The employment insurance benefit rate is set at 45% of earnings for  $i = 20$  to 64.<sup>8</sup> Employment uncertainty was modeled by using the age-specific unemployment rates for males between the ages of twenty and sixty-four for 1987.<sup>9</sup> Agents in the model economy are either employed or unemployed. All agents are willing to work, but those who are unemployed cannot work due to a shortage of demand, and wages do not adjust to clear the market. The probability of being unemployed in a given period for an agent is unrelated to the probability of being unemployed in any other period. The moral hazard problem resulting from the existence of an insurance program is not modeled in the exercise (see Hansen and Imrohoroglu (1992)).

### ***Pension Program***

The model takes as given that agents retire at the age of 65, whether or not a pension program exists. The pension program is of the pay-as-you-go type where current contributions are used to fund current benefits. In the steady state, total benefits will equal total contributions in each period. The pension benefit profile is derived from the average amount of monthly benefits put into play for males, in 1987, of \$305, an annual amount of \$3,660 (Health and Welfare Canada (1987)). Each successive generation earns an average lifetime wage  $(1+g)$  higher than the preceding generation, so the benefit profile is set to decline at a rate of  $(1+g)^{-1}$  after the age of 65.<sup>10</sup> The endowment level upon retirement,  $e_{65}$ , is set to the annual amount \$3,660, and the benefit rate,  $b_i$ , is set to 1 for  $i = 65$  to 94. The earnings and pension benefit profile is depicted in Figure 3. A second scenario, where the pension benefit rate is set to 2 (so that the annual amount corresponds to \$7,320 in 1987), was examined in order to quantify the impact of more

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<sup>8</sup> The average weekly benefit was approximately 45% of the average weekly earnings over 1972-1994 (Statistics Canada (1995: 50)).

<sup>9</sup> Figures from CANSIM database.

generous benefits on the model economies, benefits more in line with US Social Security.<sup>11</sup>

### ***Preferences and Technology***

The discount rate,  $\mathbf{b}$ , and the constant relative risk aversion parameter,  $\mathbf{s}$ , were set to 1.011 and 1.12, following the estimates established by Hurd (1989). The production function is a standard Cobb-Douglas with the share to capital,  $\mathbf{a}$ , set to 0.30. Depreciation is set at 0.06. The scale parameter,  $\mathbf{B}$ , is set to 0.4, to keep the optimal asset holdings within the grid specified in the appendix. In the small open-economy setting, the real interest rate is exogenously given, and is set to 0.02 for the model economies. The value for  $\mathbf{g}$ , technological progress is also set at 0.02. Table 1 presents model parameters that are fixed in the analysis.

## **IV. Model Economy Results**

The results presented in Tables 2 and 3 indicate that the public provision of retirement benefits can reduce the aggregate savings rate and increase the reliance on foreign capital in open economies (or increase the equilibrium interest rate in closed economies). The entitlement of modest benefits amounting to 14% of age 64 average labour earnings reduce the retirement savings motive and reduce the aggregate savings rate in model economy 1 by more than one percentage point, and reduce aggregate savings rate in model economy 2 by two percentage points. The amount of domestic savings and asset holdings is reduced by 23.1% and 21.7% respectively. In the case of more generous benefits, equal to 28% of age 64 average labour income, the aggregate savings rate is reduced from 6.3% to 3.7% in model economy 1, and from 11.0% to 6.8% in model economy 2. In such cases, the amount of domestic savings and asset holdings is reduced by over forty percent.

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<sup>10</sup> Pension benefits in Canada are indexed with the CPI, and so, will remain constant in real terms over an individual agent's retirement years.

<sup>11</sup> The US Social Security program replaces a larger portion of pre-retirement income, about double the replacement ratio of the Canada Pension Plan (Sabelhaus (1997)).

Sabelhaus (1997) shows that individuals in Canada must save at higher rates than their US counterparts because the US Social Security system is much more generous than Canadian programs for retirees. For an individual with average annual earnings of \$50,000, the US Social Security system would replace 30% of his or her earnings in retirement. In Canada, the replacement rate is 13%. Thus, one would expect the Canadian savings rate to be higher than that of the US. The results for the model economies with the benefit level set to 1 are indicative of the effect of Canadian retirement income policy on the aggregate savings rate, whereas the results with the benefit level set to 2 are indicative of the effect of US retirement income policy.

This paper has demonstrated that a public pay-as-you-go pension system can reduce the savings rate and domestic capital holdings, and increase the current account deficit and foreign indebtedness in simple life-cycle economies of overlapping generations. In a closed economy, the rate of return on capital would increase and wages would fall. This paper does not present a model that investigates the inducement effect of pensions on retirement as discussed in Feldstein (1974). Sabelhaus (1997) presents evidence that the Canadian savings rate may also have been higher during the 1970s and 1980s as the labour force participation rate of Canadian males aged 65 and older fell much more dramatically than that in the US. An investigation into these issues will be left for future work.

## **V. Conclusion**

In the major industrialized nations, the decline in aggregate savings rates has coincided with the introduction and increased generosity of public insurance programs to help alleviate poverty. One result of this improved social consciousness has been the dramatic increase in the incomes of the elderly. The provision of social security and pension benefits has lowered poverty levels of the aged.

In the US, private savings rates increased in the 1950s as the relative economic status of the elderly fell. The savings decline in the past thirty years has coincided with a period when the relative incomes of the elderly have increased (Summers and Carroll (1987: 626)). The typical 70 year-old in the US today is consuming one-fifth more than

the typical 30 year-old. This is a dramatic change from the 1960s when the average 70 year-old consumed two-thirds of the average consumption of a 30 year-old (Gokhale, Kotlikoff and Sabelhaus (1996)).

Social security and retirement policy has played an instrumental role in increasing the incomes and alleviating poverty of the elderly. As the simple model economies investigated in this paper have shown, pay-as-you-go social security programs reduce private saving, and can help explain part of the savings decline experienced by most nations. It is not the intent of this paper to encourage the reduction of benefits to encourage aggregate savings, but rather to show how pension programs reduce savings incentives. Other methods of promoting domestic rates of saving have and should be pursued. The elimination of government deficits is a mighty step towards increasing the aggregate savings rate of the US and Canada. Promoting labour productivity growth is another (see Wilson (2001c)).

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## Appendix A: Steady-State Population Distribution

The age distribution of the population at a point in time is denoted by the  $(2I \times 1)$  vector  $\mathbf{m}$ , where  $\mathbf{m} = [ \mathbf{m}_{1,f} \ \mathbf{m}_{2,f} \dots \ \mathbf{m}_{1,m} \ \mathbf{m}_{2,m} \dots \ \mathbf{m}_{I,m} ]^T$ . Agents of age  $i$ , and gender  $h$  ( $f$  for females and  $m$  for males), face a probability  $s_{i,h}$  of surviving from the age of  $i$  to the age of  $i+1$  that is constant over time. The unconditional probability of surviving to age  $k$  is  $s(k) = \prod_{j=1}^{k-1} s_j$ . The vector of age and gender specific fertility rates is  $\mathbf{f}$ , and is assumed constant over time. The number of gender newborns at time  $t$  is:

$$\mathbf{m}_{1,k,t} = \sum_{i,h} \mathbf{f}_{i,h}^k \mathbf{m}_{i,h,t-1}$$

where  $k$  denotes the gender of the newborn, and  $h$  denotes the gender of the agent (birth-parent). The population of age  $i$  agents (where  $i > 1$ ) at time  $t$  is:

$$\mathbf{m}_{i,h,t} = s_{i-1,h} \mathbf{m}_{i-1,h,t-1}.$$

The law of motion of the population is represented by the matrix  $\Gamma$ , where,  $\Gamma =$

$$\begin{bmatrix} \mathbf{f}_{1,f}^f & \mathbf{f}_{2,f}^f & \dots & \mathbf{f}_{I-1,f}^f & \mathbf{f}_{I,f}^f & \mathbf{f}_{1,m}^f & \dots & \mathbf{f}_{I-1,m}^f & \mathbf{f}_{I,m}^f \\ s_{1,f}(1+\mathbf{q}_{1,f}) & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_{2,f}(1+\mathbf{q}_{2,f}) & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots \\ 0 & 0 & \dots & s_{I-1,f}(1+\mathbf{q}_{I-1,f}) & 0 & 0 & \dots & 0 & 0 \\ \mathbf{f}_{1,f}^m & \mathbf{f}_{2,f}^m & \dots & \mathbf{f}_{I-1,f}^m & \mathbf{f}_{I,f}^m & \mathbf{f}_{1,m}^m & \dots & \mathbf{f}_{I-1,m}^m & \mathbf{f}_{I,m}^m \\ 0 & 0 & \dots & 0 & 0 & s_{1,m}(1+\mathbf{q}_{1,m}) & \dots & 0 & 0 \\ \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & s_{I-1,m}(1+\mathbf{q}_{I-1,m}) & 0 \end{bmatrix}$$

so that  $\mathbf{m} = \Gamma \mathbf{m}_{-1}$ .

In order to compute the steady-state of the model economy, the steady-state age distribution of this economy must be derived. To do this, the eigenvalues and eigenvectors of the matrix describing the law of motion of the population are computed.

The largest eigenvalue of the matrix  $\Gamma$  is equal to the steady-state growth rate of the population. The steady-state age distribution of the population is defined by the product  $\mathbf{m}^* = \tilde{\Gamma}^b \mathbf{m}$  where each element of  $\tilde{\Gamma}$  is equal to the corresponding element of  $\Gamma$  scaled by its' largest eigenvalue, and where  $b$  is sufficiently large so that  $\tilde{\Gamma}^{b+1} \mathbf{m} - \tilde{\Gamma}^b \mathbf{m} \equiv (0)$ .

## Appendix B: Solution Method

In the small open economy framework the return on capital is determined on international markets. Given the steady-state age distribution, the aggregate steady-state capital stock is calculated by solving the equation that defines the return on capital. Wages are set to the marginal productivity of labour. Steady-state equilibria are computed using the following algorithm:

- (1) Given there is no aggregate uncertainty, calculate the expected value of the aggregate labour supply, the expected value of the capital stock, total output, and the wage rate. Then, given the insurance benefit rate, and the expected number of unemployed agents and the number of pensioners of each age cohort, calculate the premium rate.
- (2) Guess the value for the transfer of accidental bequests,  $\tilde{T}$ .
- (3) Calculate the optimal decision rules,  $\tilde{c}$ ,  $\tilde{a}'$ , of all agents aged  $I_w$  (15) to  $I$  (94), with the values of  $\tilde{a}_{I_w-1}$  and  $\tilde{a}_{I+1}$  set to zero, and aggregate over the model economy. To do this, the individual's dynamic programming problem must be solved:

$$V(\tilde{a}, l, i) = \max_{(\tilde{c}, \tilde{a}')} u(\tilde{c}) + \mathbf{b}(1+g)^{1-s} s(i) E[V(\tilde{a}', l', i+1) | (\tilde{a}, l, i)],$$

subject to,

- (a)  $\tilde{c} + \tilde{a}'(1+g) \leq R\tilde{a} + (1-p)\tilde{w}l(i)e(i) + b\tilde{w}(1-l(i))e(i) + \tilde{T}$
- (b)  $\tilde{c} \leq 0, \tilde{a}' \leq 0, \tilde{a} = 0$  if  $i = I_w-1$  or  $I$ .
- (c)  $l(i) = 1$  with prob =  $p_i^e$  and 0 with prob  $1-p_i^e$ .

The decision rules are calculated by maximizing the value function on gridpoints defined over the state space. 2000 evenly spaced gridpoints were used for the asset variables  $a$  and  $a'$ , and two gridpoints were used for  $l$ . For each period of an individual's life, the value function was calculated on eight million gridpoints. The optimal value of  $a'$  for a given value of  $a$  and  $l$  was that value where the value function was maximized.

- (4) Construct population samples with random labour characteristics using the age-specific labour endowments and employment probabilities. Calculate the asset holdings of each agent of the sample, given his employment history. For each population sample, calculate the value of accidental bequests,  $\tilde{T}$ , from a random sample of agents from each cohort who die within in a given period, for each population sample, and find the average over samples.
- (5) If the average value of  $\tilde{T}$  calculated in step (4) differs from the initial guess in step (1), repeat the procedure using the result from step (4) as the initial guess until convergence is achieved.

The population samples were constructed using the steady-state population distribution for the model economy. Thirty samples of fifty thousand agents were used to find the average values in step (4).

**Table 1: Fixed Model Parameters**

<i>a</i>	<i>b</i>	<i>d</i>	<i>s</i>	<i>r</i>	<i>g</i>	<i>I<sub>w</sub></i>	<i>I<sub>w</sub></i>	<i>I</i>	<i>b<sub>20-64</sub></i>	<i>s</i>
0.3	1.011	0.06	1.12	0.02	0.02	20	65	94	0.45	1987 rates, Canada

**Table 2: Model Economy 1 Results**

	1987 fertility rates, $n = -0.79$ (all rates in percentages)		
EI Premium Rate		3.4	
Pension Benefit Rate ( $b_{65-94}$ )	0	100	200
Pension Premium Rate	0	4.7	9.4
Net Savings Rate	6.3	5.0	3.7
Foreign Capital Inflow Rate	-1.9	-0.5	0.9
Percentage of domestic asset holdings reduced due to public pensions	0	23.1	43.8

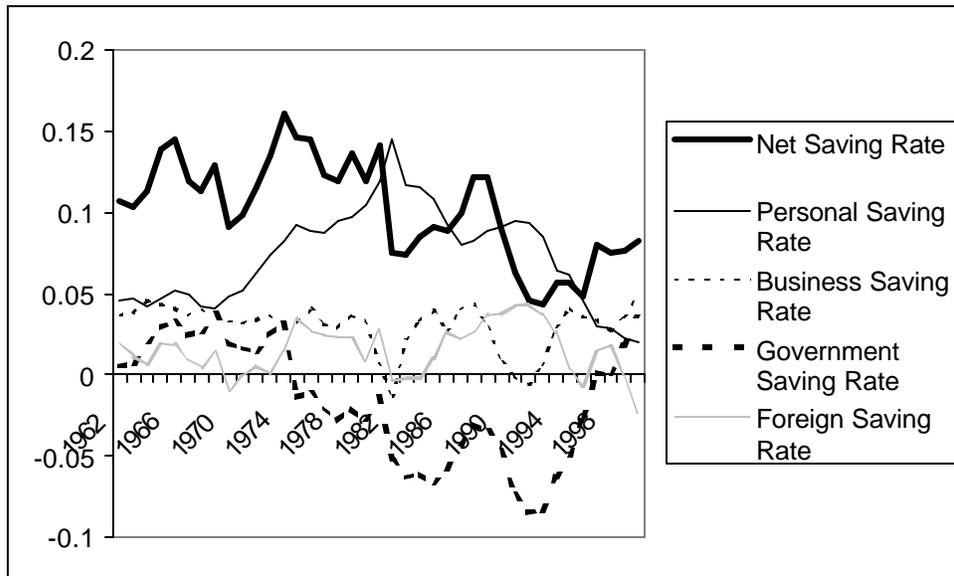
Note: The total premium rate ( $p$ ) is the sum of the EI premium rate and the pension premium rate.

**Table 3: Model Economy 2 Results**

	1970 fertility rates, $n = 0.40$ (all figures in percentages)		
EI Premium Rate		3.5	
Pension Benefit Rate ( $b_{65-94}$ )	0	100	200
Pension Premium Rate	0	3.2	6.5
Net Savings Rate	11.0	9.1	6.8
Foreign Capital Inflow Rate	-2.1	0.3	2.5
Percentage of domestic asset holdings reduced due to public pensions	0	21.7	41.1

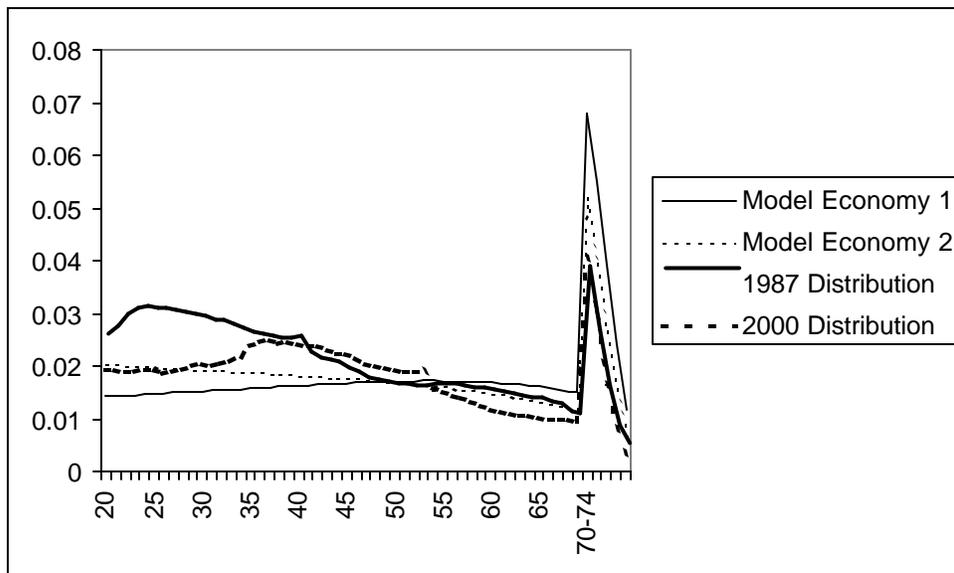
Note: The total premium rate ( $p$ ) is the sum of the EI premium rate and the pension premium rate.

**Figure 1: Canadian Net Savings Rates, 1962-2000**



Source: CANSIM Matrices 8614, 8629.

**Figure 2: Steady-State Population Distributions**



Note: The age cohorts are grouped in one year age cohorts until 70, and are then grouped in five-year age cohorts from 70 to 89, and the final age cohort consists of those aged 90 and older.

**Figure 3: Model Economy Earnings and Pension Benefit Profile**

