# HONOURS SEMINAR

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# Null Vectors, Schur-complements, and Parter Vertices

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## March 11, 2022 2:30 pm

#### **Abstract:**

One of the most important historical contributions to the inverse eigenvalue problem associated with trees is the celebrated Parter-Wiener Theorem. This theorem states if T is a tree with an associated real symmetric matrix, A, and  $m_A(\lambda) \ge 2$ , then there exists some vertex  $v \in T$  that, upon deletion from T, results in  $m_{A(v)}(\lambda) = m_A(\lambda) + 1$ for the eigenvalue  $\lambda$ . In this talk, we offer an alternate elementary proof of this seminal result utilizing the basic matrix tool known as the Schur-complement in connection with analyzing the null space structure of matrices whose graph is a tree. Moreover, the techniques used in our proof can be applied to an extension of this result by only requiring  $\lambda \in \sigma(A) \cap \sigma(A(v))$  to obtain a significant generalization of the classical Parter-Wiener theorem.



