Mathematics 103 (Applied Calculus I) Laboratory Manual

Department of Mathematics & Statistics
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1st Edition

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Inspiring Quote

> “You could say this advice is priceless”, Miss Tick said. “Are you listening?”
> “Yes”, said Tiffany.
> “Good. Now ... if you trust in yourself...”
> “Yes?”
> “... and believe in your dreams...”
> “Yes?”
> “... and follow your star...” Miss Tick went on.
> “Yes?”
> “... you’ll still get beaten by people who spent their time woking hard and learning things and weren’t so lazy. Goodbye.”

From Terry Pratchett’s *The Wee Free Men*

Module 1

Equations and Functions

1.1 Linear Functions

Answers: Page 24

1-3: Determine an equation for the line that satisfies each of the conditions below.

1. Contains the points $(-2, 1)$ and $(4, -1)$.
2. Has a slope of 5 and crosses the y-axis at $y = -12$.
3. Has a slope of -2 and contains the point $(2, 1)$.

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4. On a single coordinate plane:

   i) plot the three linear functions described below,
   ii) label both the x-intercept and the y-intercept for each line,
   iii) rank the functions by how fast they are increasing,
   iv) determine and label all the points where the lines intersect.

   (a) $f(x) = 3x - 4$.
   (b) $g(x)$ with $g(1) = 5$ and $g(3) = 4$.
   (c) $h(x)$ with slope equal to 2 and with $h(5) = 3$.

1.2 Piecewise Defined Functions

5-8: Express each of the following scenarios as a function. Sketch each function and determine whether or not each function is continuous.

5. In Sunnydale, fines for speeding depend on how fast you are traveling over the posted speed limit. Regardless of speed, each fine has a base fee of \$50. If you are caught speeding up to 10 km/h over the limit, you pay \$3.00 for each km/h over the limit; if between 10 and 20 km/h the fine increases to \$5.00 for each km/h over the limit. Between 20 and 40 km/h over your fine will be \$7.50 for each km/h over the limit.

6. In Whoville, your income tax is assessed as follows. If your income is at most \$40,000, you pay 15% of your income; from there any income up to \$80,000 is taxed at 20% and anything above \$80,000 is taxed at 25%.

7. In Cloud City, taxes are structured as follows. If you earn up to \$50,000 your taxes are 10% of your income. If you earn between \$50,000 and \$100,000 you pay a base tax of \$5000 plus 12% of your income over \$50,000. If you earn above \$100,000 you pay a base fee of \$10,000 plus 15% on your income over \$100,000.

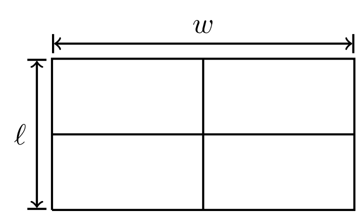
8. In the previous exercise, consider what happens if the base tax of \$5000 was changed to \$7000 or to \$3000? Do either of these changes seem fair? Why or why not?

1.3 Modeling

9-13: For each of the following situations determine a formula which describes it and state the values for which the formula is defined.

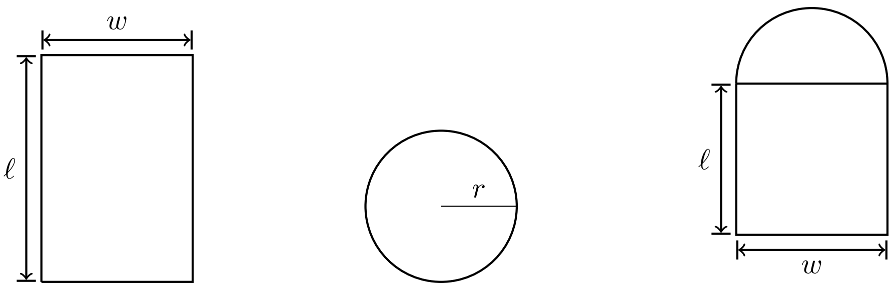
9. Let $C(x)$ represent the cost of a speeding ticket in terms of the speed x of a car in a 50 km/h zone. The cost is \$100 plus two dollars per km/h over the limit up to a speed of 75 km/h and \$150 plus five dollars for every km/h over up a speed of 100 km/h.

10. A farmer has a rectangular field with width w and length ℓ.
    (a) Determine a formula for the area of the field in terms of w and ℓ.
    (b) Determine a formula for the length of fence needed to enclose the field in terms of w and ℓ.
    (c) The farmer plans to section the field with fence as shown in the picture below. Determine the length of fencing needed for this arrangement in terms of w and ℓ.



11. A closed box has a square base of side length x and height h measured in centimeters.
    (a) Determine a formula for the volume of the box V in terms of x and h.
    (b) Determine a formula for the surface area of the box in terms of x and h.
    (c) The box is constructed from material that costs \$5/cm^2 for the top and \$3/cm^2 for the bottom and sides. Determine a formula for the cost of the box in terms of x and h.
    (d) The box is filled completely with solid gold that costs \$720/cm^3. Determine a formula for the value of the gold in the box in terms of x and h.

12. You plan to build a window and are considering three different shapes.



The glass costs \$12/cm^2 and framing around the window costs \$20/cm. Determine a formula for the total cost of each window (glass and framing) in terms of the dimensions for the shapes shown above.

13. Let $P(t)$ describe the population of bacteria at time t in hours. Find $P(t)$ if

    (a) Initially the population is 100 and doubles every hour.

    (b) Initially the population is 500 and doubles every 4 hours.

    (c) Initially the population is 1000 and grows by 10% each hour.

1.4 Domains

14-19: State the (real) values of x for which the following functions are undefined.

14. $f(x) = \dfrac{1}{x}$

15. $f(x) = \dfrac{1}{x^2 + 6x + 8}$

16. $f(x) = \dfrac{x^2 - 9}{x - 4}$

17. $f(x) = \dfrac{x - 2}{x^2 + 3}$

18. $f(x) = \dfrac{1}{\sqrt{x^2 - x}}$

19. $f(x) = \sqrt{x^2 + x + 1}$

20-23: For each of the following functions, determine all values of x for which the given function is either zero or undefined. Determine the intervals where the function is positive.

20. $f(x) = x^2 - 7x$

21. $f(x) = 24 - 4x - 4x^2$

22. $f(x) = \dfrac{x + 2}{x - 3}$

23. $f(x) = (x + 4)(x - 2)(x - 5)$

1.5 Sketching Graphs

24-26: Sketch the pairs of functions given below and determine the points where the graphs intersect.

24. $y = \frac{1}{x}$ and $y = 2x - 1$
25. $y = 2x^2 + 3x$ and $y = 6 - x^2$
26. $y = \sqrt{x+4}$ and $y = x - 2$.

1.6 Composition of Functions

27-28: Consider the following questions about composition and evaluation of functions.

27. Let $f(x) = \frac{3}{x}$ and $g(x) = x^2 + x$. Determine the following: $f(x)g(x)$, $f(g(x))$, $g(f(x))$, and $f(1)$, $g(1)$, $f(1)g(1)$, $g(f(1))$, $f(g(1))$.
28. Let $f(x) = \frac{x-1}{2x+4}$ and $g(x) = x + 2$. Determine the following: $f(x)g(x)$, $f(g(x))$, $g(f(x))$, and $f(3)$, $g(3)$, $f(3)g(3)$, $f(g(3))$.

29-32: Determine the composition $f(g(x))$ for each of the following pairs of functions.

29. $f(x) = \frac{1}{x}$ and $g(x) = x + h$.
30. $f(x) = \sqrt{x} + 3$ and $g(x) = x + h$.
31. $f(x) = x^2 + x - 4$ and $g(x) = x + h$.
32. $f(x) = \frac{x+1}{x-2}$ and $g(x) = x + h$.

33-34: Determine and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for the following functions.

33. $f(x) = 2x^2 - 4$.
34. $f(x) = \frac{x}{x+2}$.

Module 2

Exponentials and Logarithms

2.1 Exponential and Logarithm Laws

Answers: Page 27

1-4: Simplify the following expressions using the laws of logarithms and exponents.

1. $\dfrac{2^3 4^2}{2^5}$
2. $\ln(x^2 - 4x + 4)$
3. $e^{2\ln(x+3)}$
4. $\ln(e^{2x} e^3)$

5-10: Find all values of x that satisfy the following equations.

5. $e^{3x} = 4$
6. $2 + 2e^{x^2} = 36$
7. $\ln(x^2 + x) = 2$
8. $3 = \dfrac{-2}{\ln(x^2) - 3}$
9. $\ln(x-2) + \ln(x+3) = \ln(2x+6)$
10. $\dfrac{3000}{2 + 4e^x} = 80$

2.2 Applications of Exponentials and Logarithms

11-13: Consider the following questions involving compound interest.

11. Calculate the value of the following investments at the given annual interest rate after the given length of time.
    (a) An investment of \$500 at an interest rate of 6% compounding monthly for 3 years.
    (b) An investment of \$500 at an interest rate of 6% compounding continuously for 3 years.
    (c) An investment of \$1000 at an interest rate of 10% compounding quarterly for 3 years.
    (d) An investment of \$10,000 at an interest rate of 2% compounding daily for 5 years.
    (e) An investment of \$2500 at an interest rate of 10% compounding continuously for 10 years.

12. How long would it take for an investment that is continuously compounding to double when the annual interest rate is 5%?

13. What annual interest rate would be needed for an investment to double in 2 years under continuous compound interest? What interest rate would be needed for an investment to triple in 10 years under continuous compound interest?

14-15: The population of a country usually grows exponentially; this means that the population of a country at time t (years) is given by an equation of the form $P(t) = ke^{rt}$, where r is the rate of growth per year and k is the population at time $t = 0$ years.

14. The population of Nigeria in the year 2010 was $158,423,000$ and it is growing at a rate of 2.79% per year.
    (a) Determine an equation for the population of Nigeria in millions at time t using the 2010 population for $P(0)$.
    (b) What does the equation predict the population of Nigeria to be in the year 2020?
    (c) At the current rate of growth, when will the population of Nigeria reach 200,000,000?
    (d) At the current rate of growth, when will the population double the 2010 population?

15. The population of Malta in the year 1990 was only $354,170$ but had increased to $423,282$ by 2013.
    (a) Determine an equation for the population of Malta (in millions) at time t using the 1990 population for $P(0)$.
    (b) What does the equation predict the population of Malta to be in the year 2020?
    (c) According to the equation, when will the population of Malta be twice what it was in the year 1990?
    (d) At the current rate of growth, when will the population of Malta reach the 2010 population of Nigeria?

Module 3

Limits

3.1 Calculating Limits

Answers: Page 28

1-7: Evaluate the following limits.

1. $\lim_{x\to 3} \dfrac{x^2+4}{x+2}$
2. $\lim_{x\to -2} \dfrac{x^2+4x+4}{x+2}$
3. $\lim_{x\to 0} \dfrac{2-\sqrt{4-x}}{x}$
4. $\lim_{x\to 3} |x-3|$
5. $\lim_{x\to -2} \dfrac{x^2-3x-10}{x^2-4}$
6. $\lim_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}$
7. $\lim_{x\to 1} \dfrac{x-\sqrt{x}}{\sqrt{x}-1}$

3.2 Limits at Infinity

8-11: Determine the following limits at infinity.

8. $\lim_{x\to -\infty} 3e^x$
9. $\lim_{x\to -\infty} \dfrac{1}{2-4e^{2x}}$
10. $\lim_{x\to \infty} \ln\left(\dfrac{1}{x}\right)$
11. $\lim_{x\to \infty} \ln(x^2-x)$

12-15: Determine the following limits. State the degree of the numerator and denominator of each rational function.

12. $\lim_{x\to \infty} \dfrac{3-x^{26}}{x^5+2}$
13. $\lim_{x\to -\infty} \dfrac{3-x^{26}}{x^5+2}$
14. $\lim_{x\to \infty} \dfrac{x^{45}+x^{32}}{2x^{45}-3}$
15. $\lim_{x\to \infty} \dfrac{3x^4+7x^3}{x^{15}-3x^4}$

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16. Consider $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

    (a) If the degree of $p(x)$ is strictly smaller than the degree of $q(x)$ what is the limit of $f(x)$ as x goes to infinity or negative infinity?

    (b) If the degree of $p(x)$ is strictly larger than the degree of $q(x)$ what are the two values the limit of $f(x)$ can have as x goes to infinity or negative infinity?

    (c) How may you determine the limit as x goes to infinity or negative infinity if the degree of $p(x)$ equals the degree of $q(x)$?

3.3 Sketching Graphs with Limits

17-22: For each of the following functions:

(a) Determine the limit at infinity and at negative infinity,

(b) Determine all vertical asymptotes,

(c) Determine any intercepts, and

(d) Sketch the graph of the function.

17. $f(x) = \dfrac{x+3}{x^2+4}$

18. $f(x) = \dfrac{x^3-9x}{x^2+1}$

19. $f(x) = \dfrac{x^2-4}{2x^2+4}$

20. $f(x) = \dfrac{3x+3}{2x-4}$

21. $f(x) = \dfrac{x^2-2x+1}{2x^2-2x-12}$

22. $f(x) = \dfrac{5x^2-3x+1}{x^2-16}$

Module 4

Differentiation

4.1 Difference Quotients

Answers: Page 32

1-3: Determine the composition $f(g(x))$ for each of the following pairs of functions:

1. $f(x) = \dfrac{1}{x} + \sqrt{x} + 3$ and $g(x) = x + h$
2. $f(x) = x^2 + x - 4$ and $g(x) = x + h$
3. $f(x) = \dfrac{x+1}{x-2}$ and $g(x) = x + h$

4-9: Determine the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for each of the following functions.

4. $f(x) = x + 4$
5. $f(x) = x^3 - x + 4$
6. $f(x) = \dfrac{3x}{1-2x}$
7. $f(x) = \ln(x)$
8. $f(x) = e^{2x}$
9. $f(x) = \dfrac{x^2 - 1}{x + 3}$

4.2 Derivatives from First Principles

10-13: Using the definition of the derivative, calculate the derivative of each of the following functions.

10. $f(x) = 3x^2 - x$
11. $f(x) = \dfrac{3}{1-x}$
12. $f(x) = \sqrt{2 + 4x}$
13. $f(x) = \dfrac{x}{1+x}$

14-15: The function $s(t)$ represents displacement of an object (in meters) at time t (in seconds). Determine the instantaneous rate of change of displacement at the given time.

14. $s(t) = 3t^2 - 4$ at $t = 0$ and at $t = 4$ seconds.

15. $s(t) = \dfrac{2}{t+5}$ at $t = 1$ second.

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16. Let $f(x) = x^2 + x - 1$.

    (a) Determine the equation of the tangent line of this function at $x = 4$; label this equation $t(x)$.

    (b) Calculate the value of $f(x)$ and $t(x)$ at $x = 4.1$ and $x = 4.2$. What about for $x = 6$? (The point is that for values of x near 4, the values of $f(x)$ and $t(x)$ are close, but for x further from 4 the value of $t(x)$ may be quite different than $f(x)$.)

4.3 Derivatives of Polynomials

17-20: Determine the derivative of each of the following polynomials.

17. $f(x) = 4x^5 - 2x^3 + 3$

18. $f(x) = x^{24} - x^{\frac{2}{3}} - \dfrac{1}{x}$

19. $f(x) = \dfrac{2}{\sqrt{x}}$

20. $f(x) = x^{4.6} + e^2$

21-22: Determine the first, second and third derivative of each of the following polynomials.

21. $f(x) = 20x^3 - 36x^2 + 47x + 454$

22. $f(x) = x^{-4} - \sqrt{x}$

Module 5

Derivatives of Polynomials and Optimization Problems

5.1 Optimization Problems

Answers: Page 33

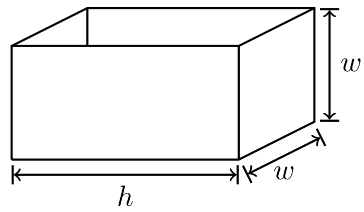
1-4: Determine the local maximums and minimums and the global maximums and minimums for each of the following functions on the given interval.

1. $f(x) = x^2 - x + 3$ on the interval $[-3, 7]$.
2. $f(x) = x^4 - x^2$ on the interval $[-2, 2]$.
3. $f(x) = 2x^3 + 9x^2 - 168x$ on the interval $[0, 10]$.
4. $f(x) = \dfrac{x^3}{1000} - \dfrac{x^2}{4}$ on the interval $[-100, 500]$.

5-14: Solve the following optimization problems.

5. Find the maximum value of xy with the constraint $x + y = 12$.
6. A farmer wishes to enclose a rectangular section of field at the edge of a straight riverbank using 1000 meters of fenceline. The longer side of the field which borders the riverbank does not require a fence. Express the area of the field enclosed by the fence as a function of the width and determine the dimensions that will maximize the enclosed area.
7. A rectangular field is to be enclosed and then divided into three equal rectangular parts using a total of 32 metres of fenceline. What are the dimensions of the field that maximize the total area of the field?
8. A closed cylindrical can has a radius r and height h measured in centimeters. Express the volume of the can in terms of the radius if the surface area must be 100π cm^2. What radius will produce a can with the greatest volume?

9. You need to design a box with no top that has a volume of 100 cm^3 as pictured below.



Determine the dimensions that require the least amount of material for construction of the box.

10. You want to build a storage shed in the shape of a box with a square base having a volume of 100 m^3. The cost of the material to make the base is \$5/m^2 and the cost of the material for the flat roof is \$3/m^2; the material for the sides will cost \$2/m^2. Determine the dimensions of the shed that will minimize the overall cost.

11. A rectangular poster contains 25 cm^2 of printed area surrounded by margins of width 2 cm on each side and 4 cm on the top and bottom. Express the total area of the poster as a function of the width of the printed section and determine the width that will maximize the total area of the poster.

12. A box with no top is to be made from a square piece of cardboard which measures 18 cm by 18 cm. To make the box, a square will be removed from each corner of the cardboard, and the resulting flaps will be folded up. What are the dimensions of the squares that produce a box with the greatest volume?

13. A cylindrical can is to hold 70π cm^3 of frozen juice. The sides are made of cardboard which costs \$0.03/cm^2; the top and bottom are made of metal that costs \$0.06/cm^2. Express the cost of constructing the can as a function of the radius of the can and determine the radius that will minimize cost.

14. A pharmacist wants to establish an optimal inventory policy for an antibiotic that requires refrigerated storage. The pharmacist expects to sell 800 units of this antibiotic at a constant rate during the next year and will place several orders of the same size equally spaced throughout the year. Each order has a delivery fee of \$16 and the carrying cost is on average \$4 per year for one package.

    (a) Let x be the order quantity and t the number of times the antibiotic is ordered in a year. Determine the inventory cost (ordering cost plus the carrying cost) in terms of x and t.

    (b) Determine the order quantity that minimizes the inventory cost and calculate the minimum inventory cost.

Module 6

Chain Rule and Related Rates

6.1 Chain Rule

Answers: Page 33

1-4: Using the chain rule, determine the derivative of each of the following functions.

1. $f(x) = (4x^{34} + 2x^{21} + x)^{-27}$
2. $f(x) = \dfrac{3}{\sqrt{x^5 + 3x^3 - 2}}$
3. $f(x) = (2 + \dfrac{1}{3x})^{12}$
4. $f(x) = (1 + \sqrt{4x})^3$

5-6: Use the chain rule to determine the following rates of change.

5. If $f(r) = r^3 + 3r^2$ and $r(t) = \sqrt{t} + t - 30$, determine $\dfrac{df}{dt}$.
6. At the Happy Unicorns' Candy Factory the cost of making c kilograms of rock candy is given by the formula $f(c) = 34c - \sqrt{c}$. The amount of candy that can be made in t hours is given by the formula $c(t) = 35t + \dfrac{2}{t}$. Determine the rate of change of the cost of making the candy over time, $\dfrac{df}{dt}$.

6.2 Implicit Differentiation

7-8: Evaluate $\dfrac{dy}{dx}$ for each of the following curves and determine the equation of the tangent line at the given point.

7. $x^2 + y^2 - 2 = y + x$ at the point $(1, 2)$.
8. $\dfrac{1}{x+y} = -x + y^2$ at the point $(-1, 4)$.

6.3 Related Rates

Recall:
The area of a circle in terms of its radius r is given by the formula $A = \pi r^2$.
The volume of a sphere in terms of its radius is given by $V = \frac{4}{3}\pi r^3$.
The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where h is the height of the cone and r is the radius of the base of the cone.

9-15: Answer the following related rates questions.

9. An oil spill takes the shape of a circle. When the spill has a radius of 50 meters its radius is growing at a rate of 0.5 m/hour. How fast is the area of the spill increasing at this time?

10. A spherical balloon is inflated at a constant rate of 5 cm^3/minute. How fast in the radius of the balloon changing when $r = 10$ cm? What about when the radius is 5 cm?

11. A 5 meter long ladder is leaning against a vertical wall. The foot of the ladder starts to slide horizontally away from the wall. At the instant the foot of the ladder is 3 m from the wall the top is falling downward at a rate of 1 m/second. How fast is the foot of the ladder moving away from the wall at this moment?

12. A cylindrical drinking straw has a radius of 0.5 cm. The straw is in a glass of water of radius 5 centimeters. Water from the glass is being sucked up the straw and the the height of the water level in the straw is increasing at a rate of 2 cm/second.
    (a) What is the rate of change of the volume of water in the straw?
    (b) At what rate is the height of the water in the glass decreasing?

13. A cone shaped cup has a height of 10 centimeters and radius of 4 cm. If coffee is being poured into the cup at a rate of 1 cm^3/s, what is the rate of change of the depth of the coffee when the depth is 5 cm? What about at 8 cm?

14. Highway 2 runs north to south and crosses Highway 1 at Moose Jaw. Dashiell is driving his car north along Highway 2, while Ada is driving east along Highway 1. At the point when Dashiell is 10 kilometers to the north of Moose Jaw, he is traveling at a speed of 80 km/h. At the same time Ada is traveling at 100km/h and is 15 kilometers to the east of Moose Jaw. At what rate is the distance between the two cars changing at this moment?

15. A plane is flying at a speed of 750 km/h and at an altitude of 5 kilometers above your head. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point that is 6 km from your position on the ground?

Module 7

Product and Quotient Rule and Graphing

7.1 Product and Quotient Rule

Answers: Page 34

1-2: Calculate the derivative of the following functions using the quotient rule.

1. $f(x) = \dfrac{1}{2x^4 - 3}$
2. $f(x) = \dfrac{5}{2x - 3}$

3-4: Calculate the derivative of the following functions using the chain rule.

3. $f(x) = \dfrac{1}{2x^4 - 3}$
4. $f(x) = \dfrac{5}{2x - 3}$

5-8: Calculate the derivative of the following functions.

5. $f(x) = (x^5 - 5x^4 + x^2 + 4)(x^6 - 4x^4 + x + 2)$
6. $f(x) = (3x + 4)^{23}(3x - 3)^{12}$
7. $f(x) = \dfrac{x^2 + 4}{x + 2}$
8. $f(x) = \dfrac{(x + 1)^2(3x - 1)^3}{x + 2}$

7.2 Sketching Asymptotes

Recall:
A function is usually **undefined** at a vertical asymptote. A function can cross a horizontal asymptote and the horizontal asymptote indicates what happens to the function as x approaches infinity or negative infinity.

9-14: Sketch a graph $f(x)$ that has a vertical asymptote $x = 1$ and a horizontal asymptote of $y = 0$ that also satisfies the given condition:

9. $f(x) > 0$ for all x.
10. $f(x) < 0$ for all x.
11. $f(x) > 0$ for $x < 1$; $f(x) < 0$ for $x > 1$.
12. $f(x) < 0$ for $x < 1$; $f(x) > 0$ for $x > 1$.
13. $f(x) < 0$ for all $x < 0$;
    $f(x) > 0$ for all $0 < x < 1$ or $x > 1$.
14. $f(x) < 0$ for all $x < 1$ or $1 < x < 2$;
    $f(x) > 0$ for all $x > 2$.

7.3 Graph Sketching

15-20: Sketch the graphs of the following functions using the following steps:

(a) Determine the x-intercepts and the y-intercept, if any.
(b) Determine any vertical asymptotes.
(c) Determine any horizontal asymptotes.
(d) Determine the intervals of increase and decrease.
(e) Determine all local maximums and minimums.
(f) Determine the intervals of concavity.
(g) Determine any inflection points.

Then produce a clear labeled sketch of the graph using the information from parts (a) to (g).

15. $f(x) = x^5 - 5x^4 + 93$
16. $f(x) = x^3 - 3x^4$
17. $f(x) = \dfrac{x^2}{x + 2}$
18. $f(x) = \dfrac{x - 3}{x^2 + 7}$
19. $f(x) = \dfrac{x^2 - 9}{x^2 + 1}$
20. $f(x) = \dfrac{(x + 3)(x - 4)}{x^2 - 4}$

Module 8

Derivatives of Exponentials and Logarithms

8.1 Derivatives with Logarithms and Exponentials

Answers: Page 40

1-4: Determine the derivatives of each of the following functions.

1. $f(x) = e^{4x^{23} - x^4}$
2. $f(x) = xe^{2x}$
3. $f(x) = \dfrac{e^{-x}}{x+1}$
4. $f(x) = (x^3 + e^x + 3)^4$

5-10: Determine the derivatives of each of the following functions.

5. $f(x) = 2\ln(x^2 - 4)$
6. $f(x) = \ln(2x)$
7. $f(x) = x\ln(x)$
8. $f(x) = (\ln(x) - 5)^3$
9. $f(x) = \dfrac{x^2 - x}{\ln(x)}$
10. $f(x) = x\ln x - (x^2 - x)e^{2x}$

11-12: Determine $\dfrac{dy}{dx}$ using logarithmic differentiation.

11. $y = x^{3x+1}$
12. $y = (2x^2 - 4)^{\frac{1}{x}}$

8.2 Applying Derivatives of Logarithms and Exponentials

13. It is canker worm season in Regina! The population of canker worms triples every week and the population continues to increase at this constant rate. On the initial day of the investation there are 50 worms in the big beautiful tree in front of my house.

(a) Determine an equation for the number of worms in my tree as a function of time t in weeks.

(b) Calculate the number of worms in my tree six weeks after the initial day of investation.

(c) At what rate is the population of worms increasing initially?

(d) At what rate is the population of works increasing six weeks after the initial day of investation?

14-17: Using the steps in Section 7.3, sketch each of the following functions.

14. $f(x) = \dfrac{10}{1+100e^{-x}}$

15. $f(x) = x\ln(x)$ for $x > 0$

16. $f(x) = (\ln(x))^2$ for $x > 0$

17. $f(x) = 10xe^x$

18-21: Optimization Problems.

18. Determine the maximum of $\ln(x) - x^2$ for $x > 0$.

19. Determine the local maximum and minimum of $f(x) = x^2e^x$.

20. The value of an investment is given by $30e^{0.05t}$ (dollars) and the investment brokers fees are determined by $\dfrac{1}{t}$ times the value of the investment at time t. At what time will the brokers fees be at a minimum? What is the fee at this time?

21. The effectiveness of studying for a calculus test depends on the number of hours a student spends studying for the test. From experimental evidence, the effectiveness of studying after t hours is given by $E(t) = \dfrac{2}{5}\left(9 + te^{\frac{-t}{10}}\right)$. A student has up to 20 hours to study for the quiz. How many hours are needed for maximum effectiveness?

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22. There is a rumour afloat at your university that your calculus professor secretly works as a clown at children's birthday parties on the weekend. At time t (in hours) the number of students that know this rumour is given by the equation

$$C(t) = \frac{10,000}{1+500e^{-\frac{t}{10}}}.$$

(a) How many students know the rumour at time $t = 0$?

(b) Everyone will eventually hear the rumour. What is the population of the university?

(c) How long until 2000 students have heard the rumour?

(d) When is the rumour spreading the fastest?

(e) Graph this function using a graphing calculator. (Functions of the form

$$f(t) = \frac{c}{1+ae^{-bt}}$$

are called *logistic functions* and they model the growth pattern of some phenomena.)

Module 9

Integration

9.1 Antiderivatives

Answers: Page 43

1-6: Determine an antiderivative for each of the following functions.

1. $f(x) = 3x^2 - 5x + 6$
2. $f(x) = \dfrac{x^3 + 4}{x^2}$
3. $f(x) = \sqrt{x} + \dfrac{2}{\sqrt{x}}$
4. $f(x) = \sqrt[3]{x^2} - 4x^6 + e^2$
5. $f(x) = \dfrac{3}{x}$
6. $f(x) = e^{2x}$

7-10: Simplify then evaluate each of the following indefinite integrals.

7. $\displaystyle\int \frac{4x^3 + 5\sqrt{x} - 4x}{x}\, dx$
8. $\displaystyle\int (x+3)(x-4)\, dx$
9. $\displaystyle\int x\sqrt{2x}\, dx$
10. $\displaystyle\int \sqrt{x}(\sqrt{x} + \sqrt{2})\, dx$

9.2 The Definite Integral

11-14: Compute each of the following definite integrals.

11. $\displaystyle\int_0^2 3x + 6\, dx$
12. $\displaystyle\int_0^3 (2x - 4)\, dx$
13. $\displaystyle\int_{-1}^{10} e^x - x\, dx$
14. $\displaystyle\int_1^{e^2} \frac{1}{x}\, dx$

9.3 The Substitution Rule

15-19: Evaluate each of the following indefinite integrals using the substitution rule.

15. $\displaystyle\int \frac{x^2+2x}{(x^3+3x^2+4)^5}\,dx$

16. $\displaystyle\int (5x+1)\sqrt{5x^2+2x}\,dx$

17. $\displaystyle\int x\sqrt{4x+1}\,dx$

18. $\displaystyle\int 2xe^{x^2-4}\,dx$

19. $\displaystyle\int \frac{4x-4}{2x^2-4x}\,dx$ (with $x>0$)

20-22: Evaluate each of the following definite integrals using the substitution rule.

20. $\displaystyle\int_0^2 x^3\sqrt{x^4+9}\,dx$

21. $\displaystyle\int_0^5 xe^{x^2}\,dx$ (with $x>0$)

22. $\displaystyle\int_1^3 \frac{x}{(2x^2+1)^2}\,dx$

9.4 Areas Under a Curve

23-25: Using definite integrals, determine answers to the following questions.

23. Calculate the area bounded by the x-axis and the y-axis and the curve $y=5-x^2$.

24. Let $g(x)=1-2x$ and $f(x)=x^2+10x+21$.
    (a) Graph $g(x)$ and $f(x)$.
    (b) Determine where $g(x)$ and $f(x)$ intersect.
    (c) Calculate the area bounded by $g(x)$ and $f(x)$.

25. Consider the curves $f(x)=e^x$ and $g(x)=20-e^x$.
    (a) Graph $g(x)$ and $f(x)$.
    (b) Determine where $g(x)$ and $f(x)$ intersect.
    (c) Calculate the area bounded by $g(x)$ and $f(x)$ and the y-axis.

9.5 Average Value of a Function

26-27: Consider the average value of the function given in each of the following situations.

26. The value of a commodity fluctuates over time and is given by the function

$$f(t) = \frac{1}{100}t^3 - 71t^2 + 1450t - 8325$$

where t is time in years. What is the average value of the commodity from $t = 0$ and $t = 60$?

27. The demand for long-stemmed roses is given by the function $f(t) = \dfrac{5000\ln(t+11) - 3}{t+11}$ where t is the number of days from January 1^{st}. What is the average demand for long stemmed roses over a year (the period from $t = 0$ to $t = 365$)?

Answers

1 Exercises (page 3)

1. $3y + x = 1$
2. $y = 5x - 12$
3. $y = -2x + 5$

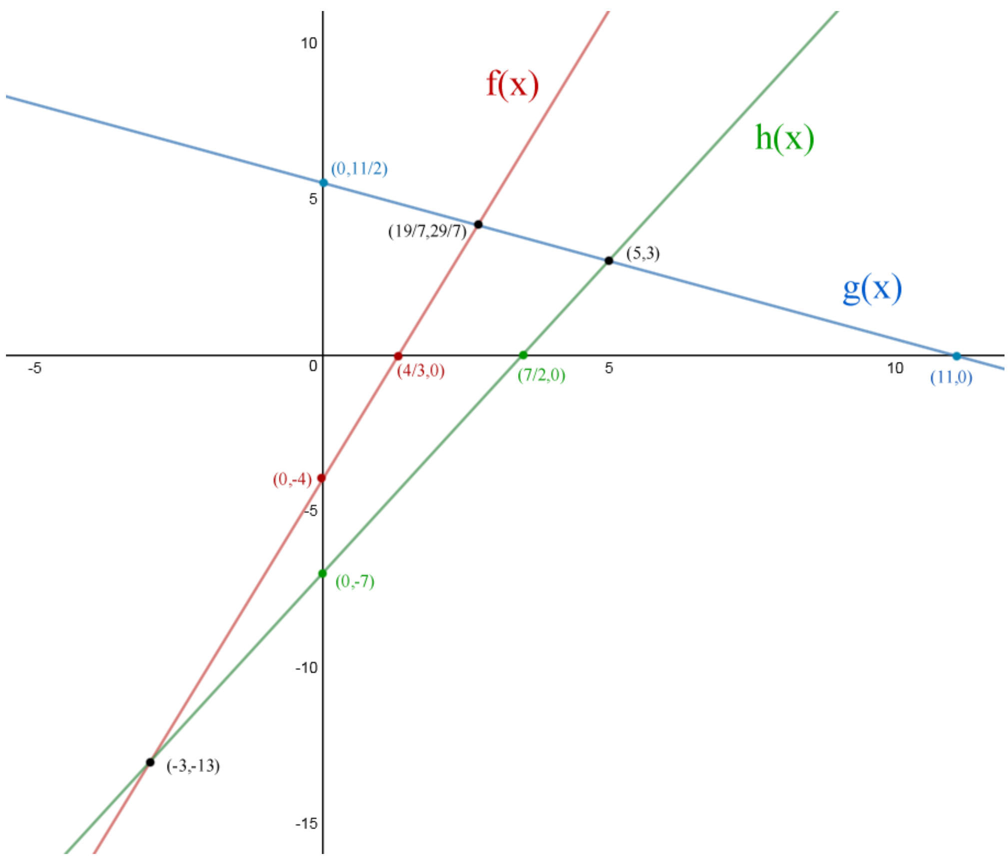
4. The linear functions have equations

   (a) $f(x) = 3x - 4$

   (b) $g(x) = -\dfrac{1}{2}x + \dfrac{11}{2}$

   (c) $h(x) = 2x - 7$

   See graph for intersection points:



5. If x represents speed then the fine (in dollars) is,
$$F(x) = \begin{cases} 0 \text{ if } x \leq 50 \\ 50 + 3(x - 50) \text{ if } 50 < x \leq 60 \\ 80 + 5(x - 60) \text{ if } 60 < x \leq 70 \\ 130 + 7.5(x - 70) \text{ if } 70 < x \leq 90 \end{cases}$$

6. If x represents your income then your taxes (in dollars) are,
$$T(x) = \begin{cases} 0.15x \text{ if } x \leq \$40,000 \\ 0.2x \text{ if } \$40,000 < x \leq \$80,000 \\ 0.25x \text{ if } \$80,000 < x \end{cases}$$

7. If x represents your income then your taxes (in dollars) are,
$$T(x) = \begin{cases} 0.10x \text{ if } x \leq \$50,000 \\ 5000 + 0.12(x - 50,000) \text{ if } \$50,000 < x \leq \$100,000 \\ 10,000 + 0.15(x - 100,000) \text{ if } \$100,000 < x \end{cases}$$

8. If the base tax changed from \$5000 the graph would be discontinuous; there would be a *jump* from one tax bracket to the next. If the base was \$7000, a person earning \$50,001 would pay an additional \$2000 compared to a person earning \$49,999. In other words, a difference in income of just two more dollars could mean a difference in your tax assessment of \$2000! Similarly if the base was \$3000.

9. $C(x) = \begin{cases} 0 \text{ if } x \leq 50 \\ 100 + 2(x - 50) \text{ if } 50 < x \leq 75 \\ 150 + 5(x - 75) \text{ if } 75 < x \leq 100 \end{cases}$

10. (a) $A = \ell w$ defined for ℓ and w positive.
    (b) $F = 2(w + \ell)$ defined for ℓ and w positive.
    (c) $F = 3(w + \ell)$ defined for ℓ and w positive.

11. (a) $V = x^2 h$ defined for x and h positive.
    (b) $SA = 2x^2 + 4xh$ defined for x and h positive.
    (c) $C = 5x^2 + 3(x^2 + 4xh)$ defined for x and h positive.
    (d) $V = 720x^2 h$ defined for x and h positive.

12. $\text{Cost} = \begin{cases} 12\ell w + 40(w + \ell) \text{ for the rectangular shape} \\ 12\pi r^2 + 40\pi r \text{ for the circular shape} \\ 12(\ell w + \dfrac{\pi w^2}{8}) + 20(2(w + \ell) + \dfrac{\pi w}{2}) \text{ for the last shape} \end{cases}$
    defined for w, ℓ and r positive.

13. (a) $P(t) = 100(2^t)$ defined for t positive.
    (b) $P(t) = 500(2^{\frac{t}{4}})$ defined for t positive.
    (c) $P(t) = 1000(1.1^t)$ defined for t positive.

14. $x = 0$

15. $x = -4$ and $x = -2$

16. $x = 4$

17. $f(x)$ is defined for all real numbers x.

18. $f(x)$ is undefined (as a real-valued function) for $0 \leq x \leq 1$

19. $f(x)$ is defined for all real numbers x.

20. $f(x)$ is defined for any real number and is zero at $x = 0$ and $x = 7$. This function is positive if $x < 0$ and if $7 < x$.

21. $f(x)$ is defined for any real number and is zero at $x = -3$ and $x = 2$. This function is positive if $-3 < x < 2$.

22. $f(x)$ is undefined at $x = 3$ and is zero at $x = -2$. This function is positive if $x < -2$ and if $3 < x$.

23. $f(x)$ is defined for any real number and is zero at $x = -4, 2$ and 5. This function is positive if $-4 < x < 2$ and if $5 < x$.

24. Intersect at $(-\frac{1}{2}, -2)$ and $(1, 1)$.

25. Intersect at $(-2, 2)$ and $(1, 5)$.

26. Intersect at $(5, 3)$.

27. $f(x)g(x) = 3(x+1), f(g(x)) = \dfrac{3}{x^2+x}, g(f(x)) = \dfrac{9}{x^2} + \dfrac{3}{x},$
    $f(1) = 3, g(1) = 2, f(1)g(1) = 6, f(g(1)) = \dfrac{3}{2}, g(f(1)) = 12$

28. $f(x)g(x) = \dfrac{x-1}{2}, f(g(x)) = \dfrac{x+1}{2x+8}, g(f(x)) = \dfrac{5x+7}{2x+4},$
    $f(3) = \dfrac{1}{5}, g(3) = 5, f(3)g(3) = 1, f(g(3)) = \dfrac{2}{7}, g(f(3)) = \dfrac{11}{5}$

29. $f(g(x)) = \dfrac{1}{x+h}$

30. $f(g(x)) = \sqrt{x+h} + 3$

31. $f(g(x)) = (x+h)^2 + (x+h) - 4$

32. $f(g(x)) = \dfrac{x+h+1}{x+h-2}$

33. $\dfrac{f(x+h) - f(x)}{h} = 2x + h$

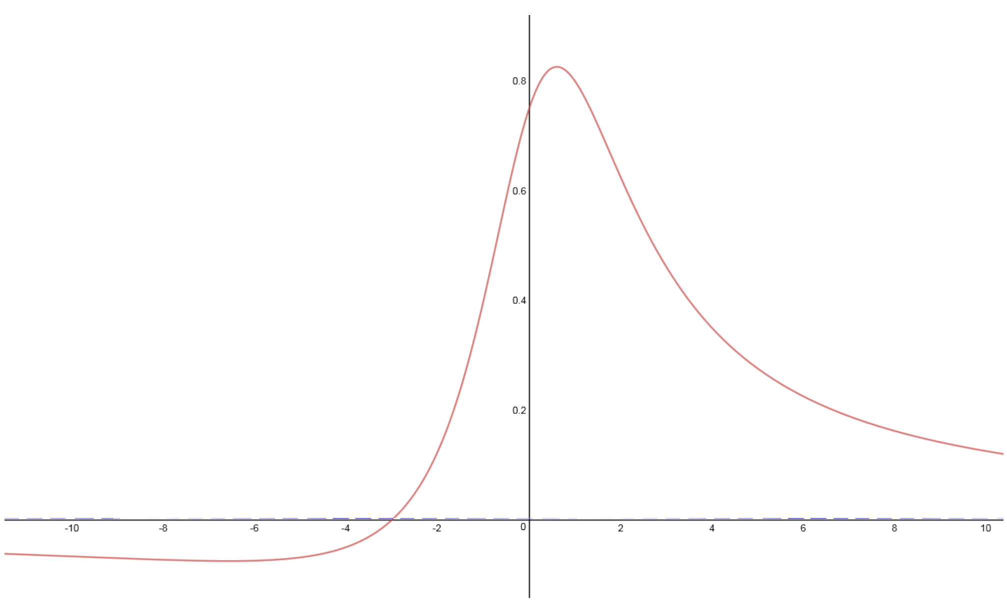
34. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2}{(x+2)(x+h+2)}$

2 Exercises (page 7)

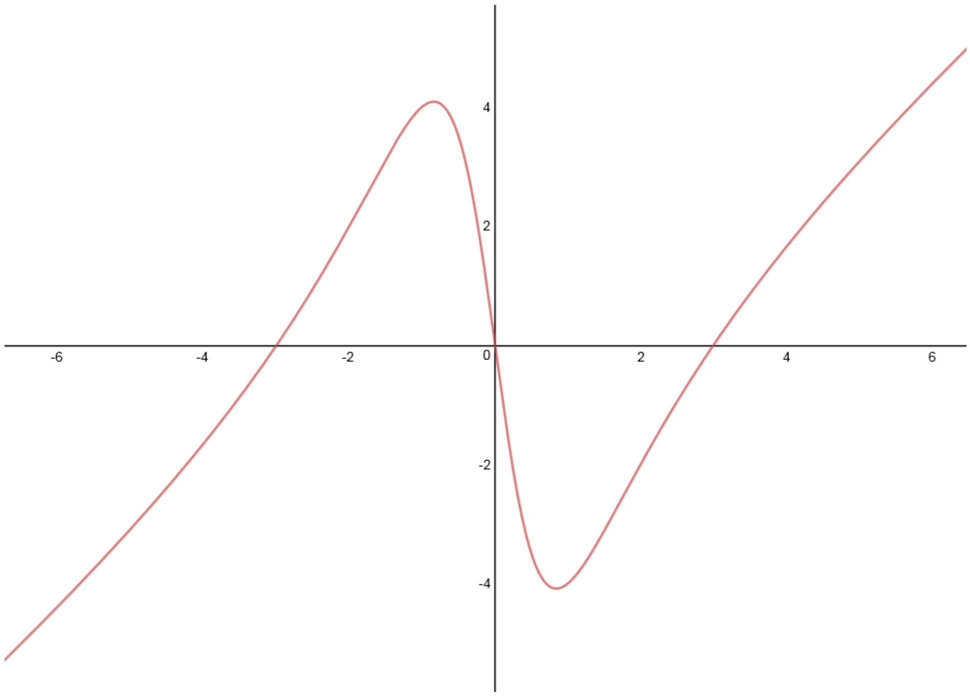
1. 4
2. $2\ln(x-2)$
3. $(x+3)^2$
4. $2x+3$
5. $x = \dfrac{\ln(4)}{3}$
6. $x = \pm\sqrt{\ln(17)} \approx \pm 2.833$
7. $x = \dfrac{-1 \pm \sqrt{1+4e^2}}{2}$, which is $x \approx 2.264$ or -3.264
8. $x = \pm e^{7/6} \approx \pm 3.211$
9. $x = 4$
10. $x = \ln\left(\dfrac{71}{8}\right) \approx 2.183$
11. (a) \$598.34
    (b) \$598.61
    (c) \$1344.89
    (d) \$11,051.68
    (e) \$6795.70
12. $t = \dfrac{\ln(2)}{0.05} = 13.86$ years
13. 34.65% and 10.98%
14. (a) $P(t) = 158.423e^{.0279t}$
    (b) $P(2020) = 209.404684$ or 209,404,684
    (c) $t = 8.36$ years from 2010
    (d) $t = 24.84$ years from 2010
15. (a) $P(t) = 0.35417e^{0.00775t}$
    (b) $P(2020) = 0.446874$ or 446,874
    (c) $t = 89.44$ years from 1990
    (d) $t = 787.52$ years from 1990

3 Exercises (page 9)

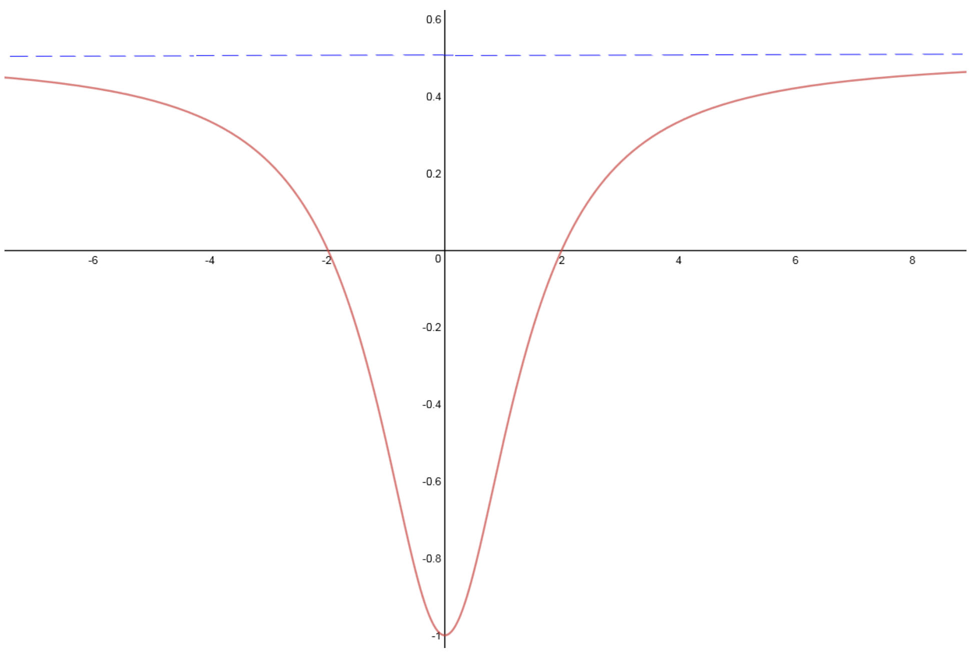
1. $\frac{13}{5}$
2. 0
3. $\frac{1}{4}$
4. 0
5. $\frac{7}{4}$
6. $-\frac{1}{9}$
7. 1
8. 0
9. $\frac{1}{2}$
10. $-\infty$
11. ∞
12. Degree of the numerator is 26; the denominator is of degree 5. The limit is $-\infty$.
13. Degree of the numerator is 26; the denominator is of degree 5. The limit is ∞.
14. Degree of the numerator and denominator is 26. The limit is $\frac{1}{2}$.
15. Degree of the numerator is 4; the denominator is of degree 15. The limit is 0.
16. (a) The limit will be zero.
    (b) The limit is either infinity or negative infinity.
    (c) The limit will be the quotient of the coefficient of the term with the highest power for $p(x)$ and the coefficient of the term with the highest power for $q(x)$.
17. (a) $\lim_{x\to\infty} f(x) = 0$ $\qquad$ $\lim_{x\to-\infty} f(x) = 0$
    (b) No vertical asymptotes; Horizontal asymptote $y = 0$
    (c) Intercepts at $\left(0, \frac{3}{4}\right)$ and $(-3, 0)$
    (d)



18. (a) $\lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to-\infty} f(x) = -\infty$
    (b) No vertical or horizontal asymptotes.
    (c) Intercepts at $(0,0)$ and $(\pm 3, 0)$
    (d)



19. (a) $\lim_{x\to\infty} f(x) = \dfrac{1}{2} \qquad \lim_{x\to-\infty} f(x) = \dfrac{1}{2}$
    (b) No vertical asymptotes; Horizontal asymptote $y = \dfrac{1}{2}$
    (c) Intercepts at $(\pm 2, 0)$
    (d)



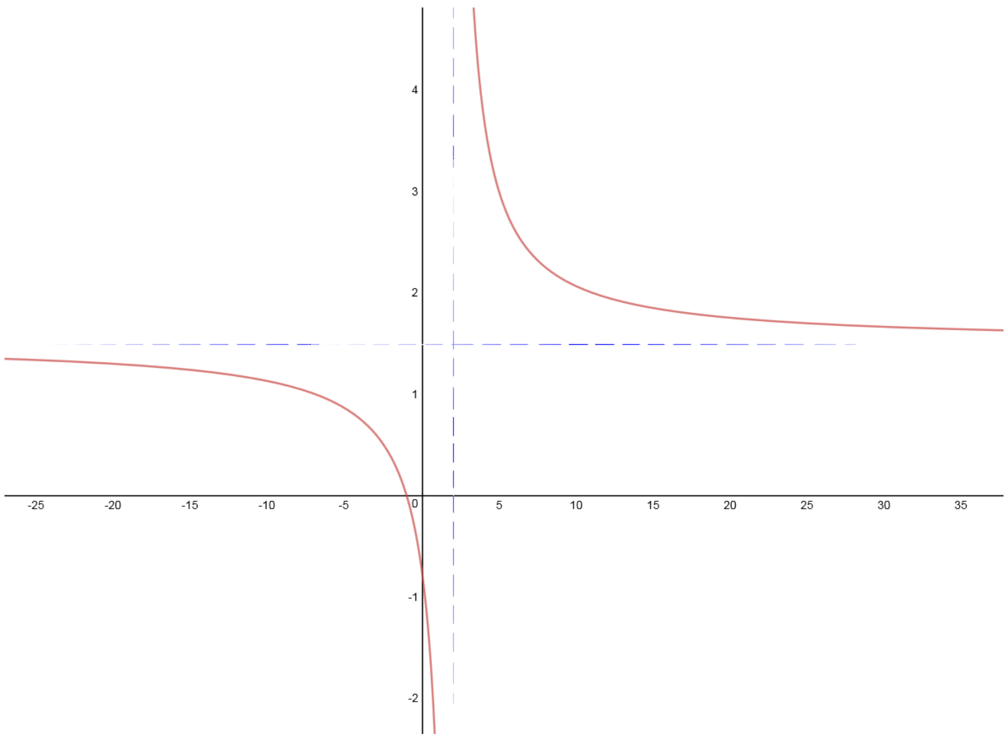
20. (a) $\lim_{x\to\infty} f(x) = \dfrac{3}{2}$ $\qquad \lim_{x\to-\infty} f(x) = \dfrac{3}{2}$

(b) $f(x)$ is undefined at $x = 2$ $\qquad \lim_{x\to 2^-} f(x) = -\infty$ $\qquad \lim_{x\to 2^+} f(x) = \infty$

Vertical asymptote $x = 2$; Horizontal asymptote $y = \dfrac{3}{2}$

(c) Intercepts at $\left(0, -\dfrac{3}{4}\right)$ and $(-1, 0)$

(d)



21. (a) $\lim_{x\to\infty} f(x) = \dfrac{1}{2}$ $\qquad \lim_{x\to-\infty} f(x) = \dfrac{1}{2}$

(b) $f(x)$ is undefined at $x = -2, 3$

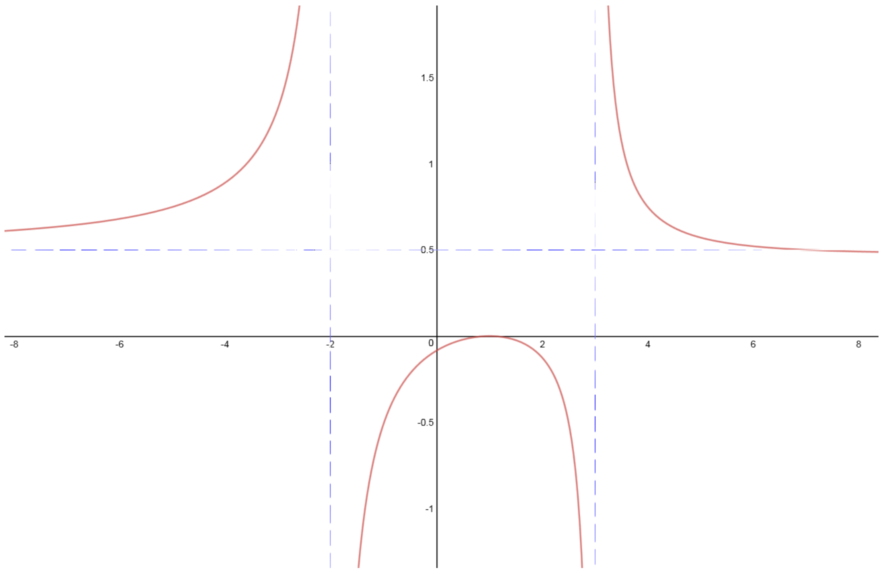
$\lim_{x\to -2^-} f(x) = \infty$ $\qquad \lim_{x\to -2^+} f(x) = -\infty$

$\lim_{x\to 3^-} f(x) = -\infty$ $\qquad \lim_{x\to 3^+} f(x) = +\infty$

Vertical asymptotes $x = -2$ and $x = 3$; Horizontal asymptote $y = \dfrac{1}{2}$

(c) Intercepts at $\left(0, -\dfrac{1}{12}\right)$ and $(1, 0)$

(d)



22. (a) $\lim_{x\to\infty} f(x) = 5 \qquad \lim_{x\to-\infty} f(x) = 5$

(b) $f(x)$ is undefined at $x = \pm 4$

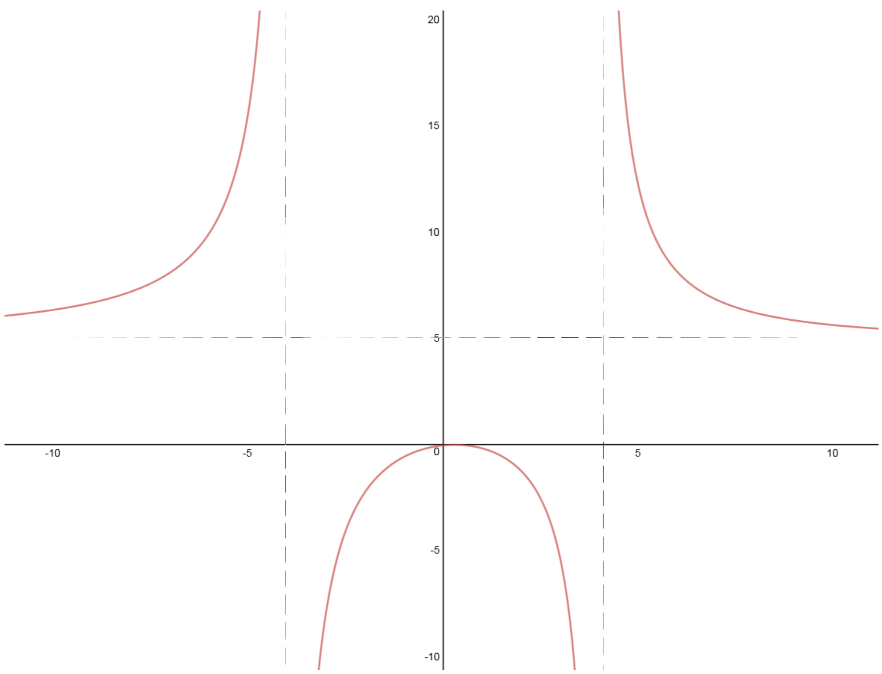
$\lim_{x\to-4^-} f(x) = \infty \qquad \lim_{x\to-4^+} f(x) = -\infty$

$\lim_{x\to4^-} f(x) = -\infty \qquad \lim_{x\to4^+} f(x) = +\infty$

Vertical asymptotes $x = \pm 4$; Horizontal asymptote $y = 5$

(c) Intercept at $\left(0, -\dfrac{1}{16}\right)$

(d)



4 Exercises (page 11)

1. $f(g(x)) = \dfrac{1}{x+h} + \sqrt{x+h} + 3$
2. $f(g(x)) = x^2 + 2xh + h^2 + x + h - 4$
3. $f(g(x)) = \dfrac{x+h+1}{x+h-2}$
4. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h+4)-(x+4)}{h} = 1$
5. $\dfrac{f(x+h)-f(x)}{h} = 3x^2 + 3xh + h^2 - 1$
6. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{3}{(1-2x-2h)(1-2x)}$
7. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\ln(x+h)-\ln(x)}{h}$
8. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{e^{2x}(e^{2h}-1)}{h}$
9. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{x^2+xh+6x+3h+1}{(x+h+3)(x+3)}$
10. $f'(x) = 6x - 1$
11. $f'(x) = \dfrac{3}{(1-x)^2}$
12. $f'(x) = \dfrac{2}{\sqrt{2+4x}}$
13. $f'(x) = \dfrac{1}{(1+x)^2}$
14. $s'(0) = 0$ m/s and $s'(4) = 24$ m/s
15. $s'(1) = -\dfrac{1}{18}$ m/s
16. (a) $t(x) = 9x - 17$
    (b) $f(4.1) = 19.91$, $t(4.1) = 19.9$, $f(4.2) = 20.84$, $t(4.2) = 20.8$, $f(6) = 41$, $t(6) = 37$
17. $f'(x) = 20x^4 - 6x^2$
18. $f'(x) = 24x^{23} - \dfrac{2}{3}x^{-\frac{1}{3}} + \dfrac{1}{x^2}$
19. $f'(x) = -x^{-3/2}$
20. $f'(x) = 4.6x^{3.6}$

21. $f'(x) = 60x^2 - 72x + 47$
    $f''(x) = 120x - 72$
    $f'''(x) = 120$

22. $f'(x) = -4x^{-5} - \frac{1}{2}x^{-\frac{1}{2}}$
    $f''(x) = 20x^{-6} + \frac{1}{4}x^{-\frac{3}{2}}$
    $f'''(x) = -120x^{-7} - \frac{3}{8}x^{-\frac{5}{2}}$

5 Exercises (page 13)

1. Local and global min at $x = 0.5$ and global max at $x = 7$.
2. Local and global min at $x = \pm\frac{1}{\sqrt{2}}$. Local max at $x = 0$, global max at $x = \pm 2$.
3. Local and global min at $x = 4$, global max at $x = 10$.
4. Local max at $x = 0$, local min at $x = \frac{500}{3}$. Global max at $x = 500$.
5. Maximum of $xy = 36$ when $x = y = 6$.
6. The area $A(w) = 1000w - 2w^2$ is maximized when $w = 250$ m and $\ell = 500$ m.
7. $w = 4$ m and $\ell = 8$ m
8. $v = \pi r(50 - r^2)$, $r = \sqrt{\frac{50}{3}} \approx 4.082$ cm
9. $w = \sqrt[3]{50} \approx 3.68$ cm and $h \approx 7.37$ cm
10. The same solution as the previous question; $3.68 \times 3.68 \times 7.37$ meters.
11. $A = 8w + 57 + \frac{100}{w}$, maximized at $w = \sqrt{12.5} \approx 3.54$ cm
12. Squares of side length 3 cm.
13. $C(r) = 0.12\pi r^2 + \frac{4.2\pi}{r}$, and minimized at $r = \sqrt[3]{17.5} \approx 2.6$ cm.
14. (a) Inventory Cost $= \$16t + \$4\left(\frac{x}{2}\right)$
    (b) The cost is minimized at $t = 10$ and $x = 80$.

6 Exercises (page 15)

1. $f'(x) = -27(4x^{34} + 2x^{21} + x)^{-28}(136x^{33} + 42x^{20} + 1)$
2. $f'(x) = -\frac{3}{2}(x^5 + 3x^3 - 2)^{-3/2}(5x^4 + 9x^2)$
3. $f'(x) = 12(2 + \frac{1}{3x})^{11}(-\frac{1}{3}x^{-2})$

4. $f'(x) = 3(1+\sqrt{4x})^2(\frac{1}{2}(4x)^{-1/2}(4))$

5. $\frac{df}{dt} = (3(\sqrt{t}+t-30)^2 + 6(\sqrt{t}+t-30))(\frac{1}{2\sqrt{t}}+1)$

6. $\frac{df}{dt} = (34 - \frac{1}{2\sqrt{35t+\frac{2}{t}}})(35 - \frac{2}{t^2})$

7. $\frac{dy}{dx} = \frac{1-2x}{2y-1}$ and $\frac{dy}{dx} = -\frac{1}{3}$ at $x = 1$ and $y = 2$. An equation for the tangent line is $y - 2 = -\frac{1}{3}(x-1)$.

8. $\frac{dy}{dx} = \frac{(x+y)^2-1}{1+2y(x+y)^2}$ and $\frac{dy}{dx} = 8/73$ at $x = -1$ and $y = 4$. An equation for the tangent line is $y - 4 = \frac{8}{73}(x+1)$.

9. 50π m^2/hour

10. At 10 cm, $\frac{dr}{dt} = \frac{\pi}{80}$ cm/min; at 5cm, $\frac{dr}{dt} = \frac{\pi}{20}$ cm/min

11. $\frac{4}{3}$ m/s

12. (a) $\frac{dV}{dt} = 0.5\pi$ cm/sec

    (b) $\frac{dh}{dt} = -0.02$ cm/sec

13. 0.08 cm/s, 0.03 cm/s

14. 127.58 km/h

15. 576.17 km/h

7 Exercises (page 17)

1. $f'(x) = \frac{-8x^3}{(2x^4-3)^2}$

2. $f'(x) = \frac{-10}{(2x-3)^2}$

3. $f'(x) = -(2x^4-3)^{-2}(8x^3)$

4. $f'(x) = -5(2x-3)^{-2}(2)$

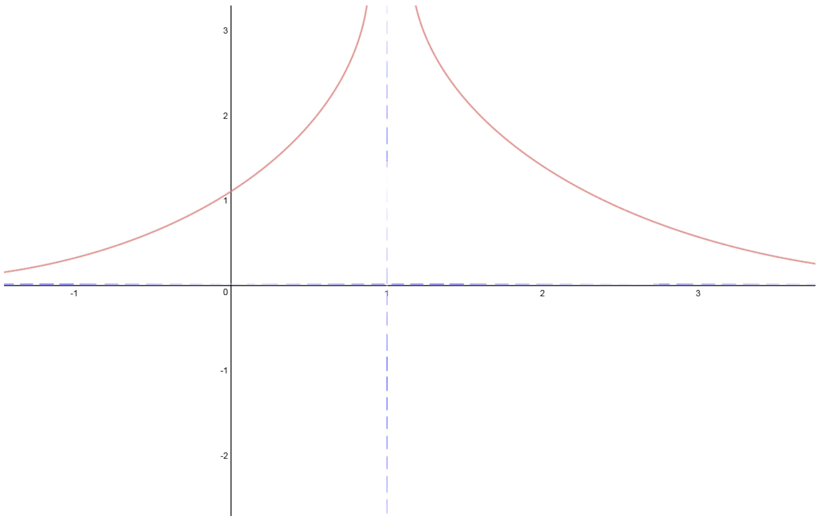
5. $f'(x) = (5x^4-20x^3+2x)(x^6-4x^4+x+2) + (x^5-5x^4+x^2+4)(6x^5-16x^3+1)$

6. $f'(x) = 23(3x+4)^{22}(3)(3x-3)^{12} + (3x+4)^{23}(12)(3x-3)^{11}(3)$

7. $f'(x) = \frac{(x^2+4)-(x+2)(2x)}{(x+2)^2}$

8. $f'(x) = \frac{[2(x+1)(3x-1)^3 + (x+1)^2(3)(3x-1)^2(3)](x+2) - (x+1)^2(3x-1)^3}{(x+2)^2}$

9.



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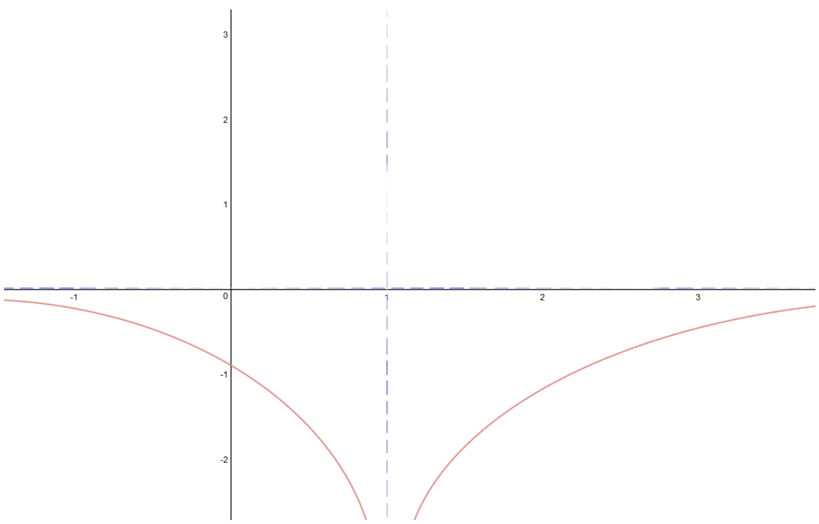

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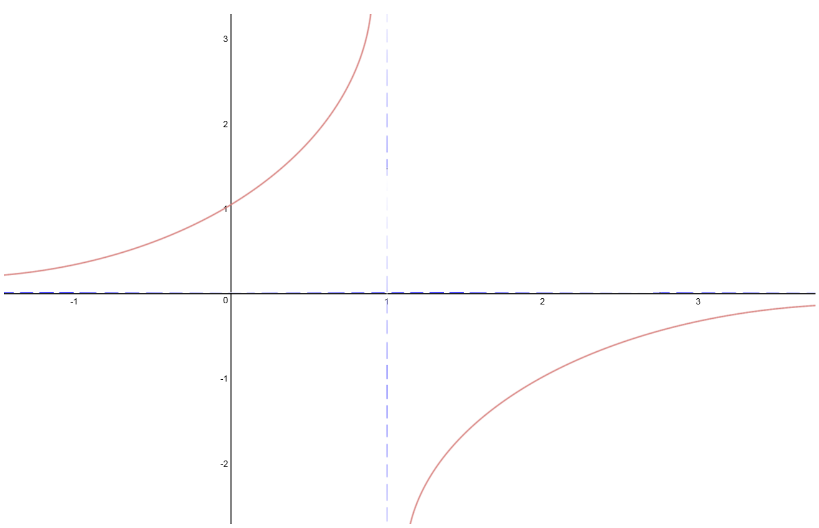

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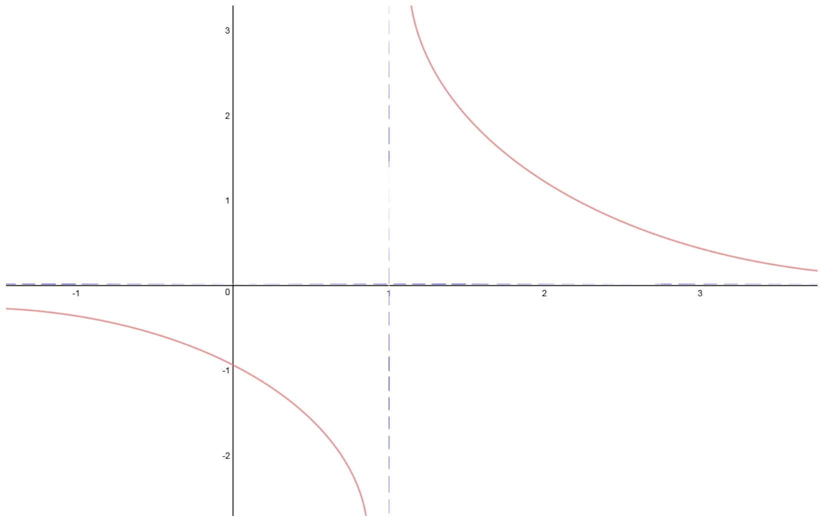

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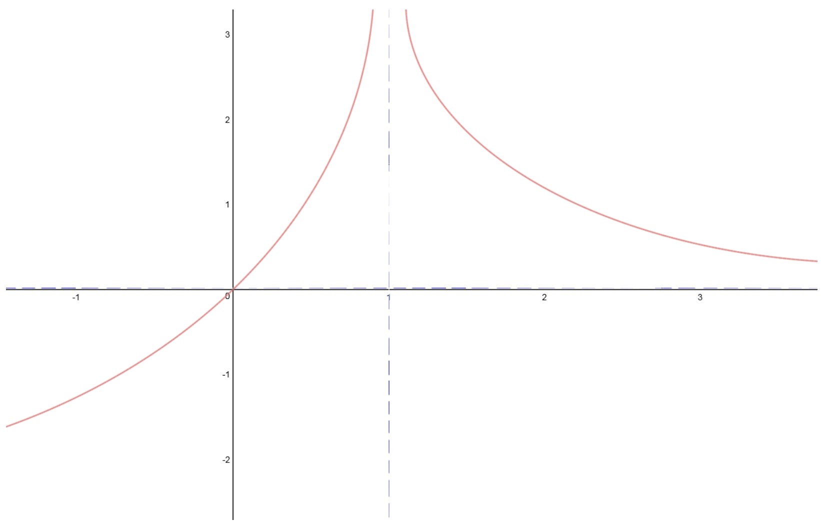

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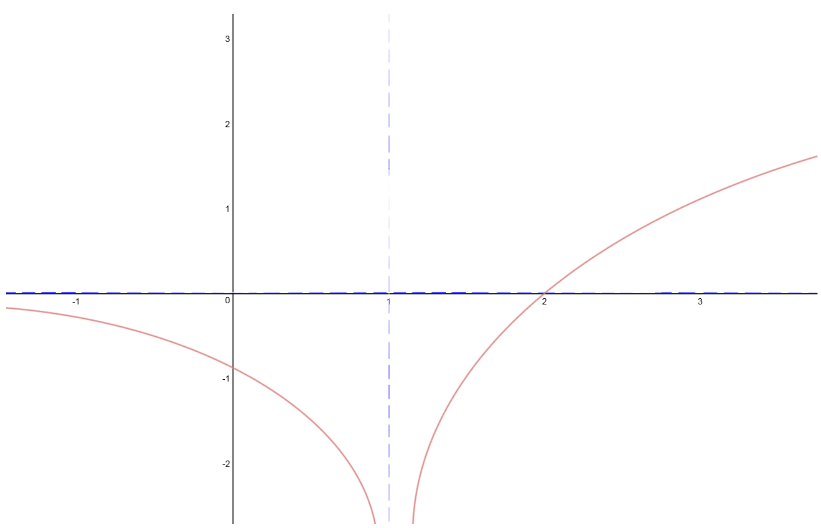
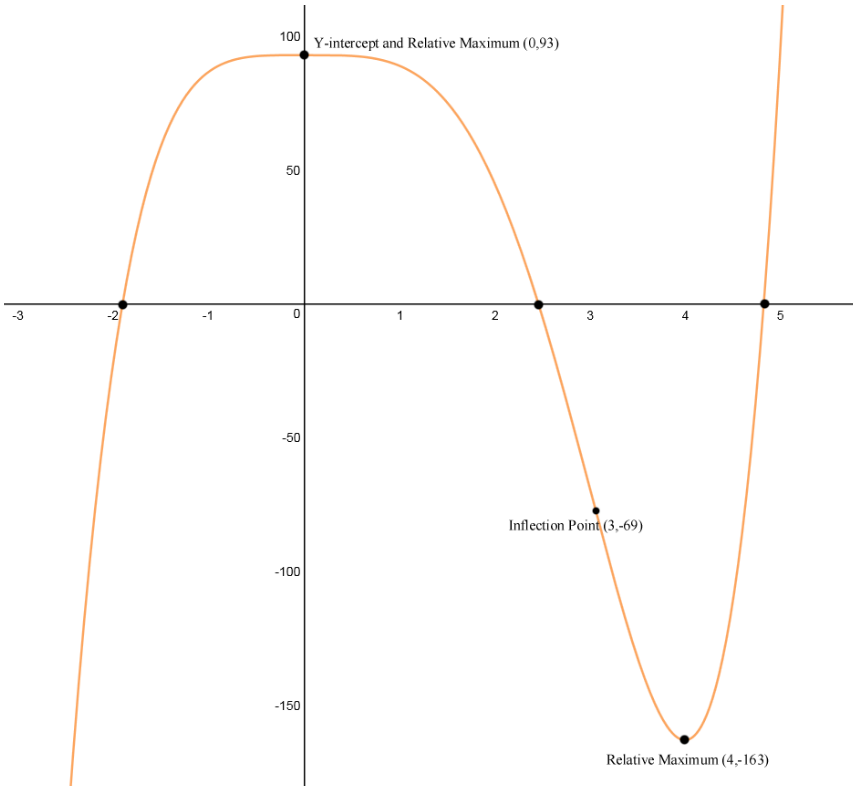
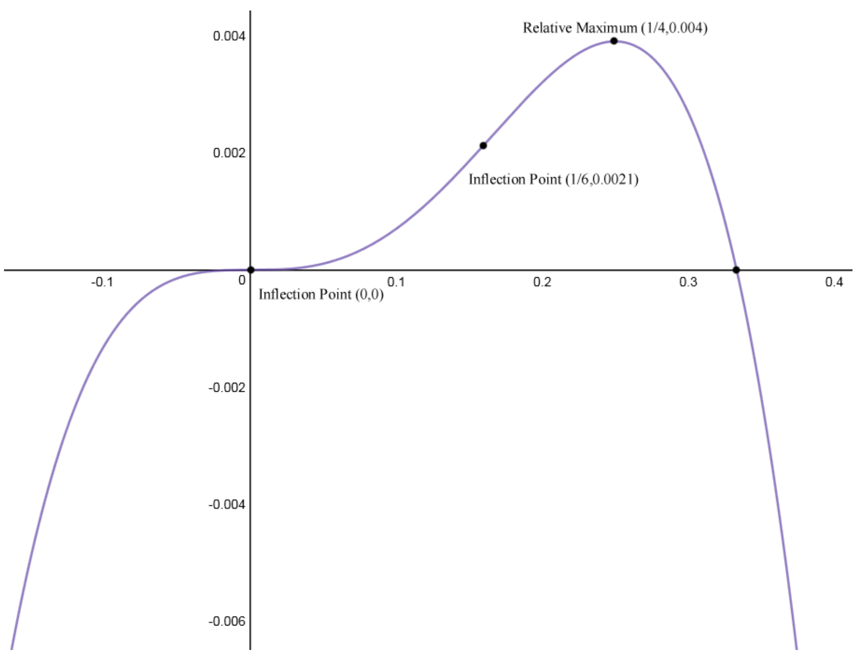


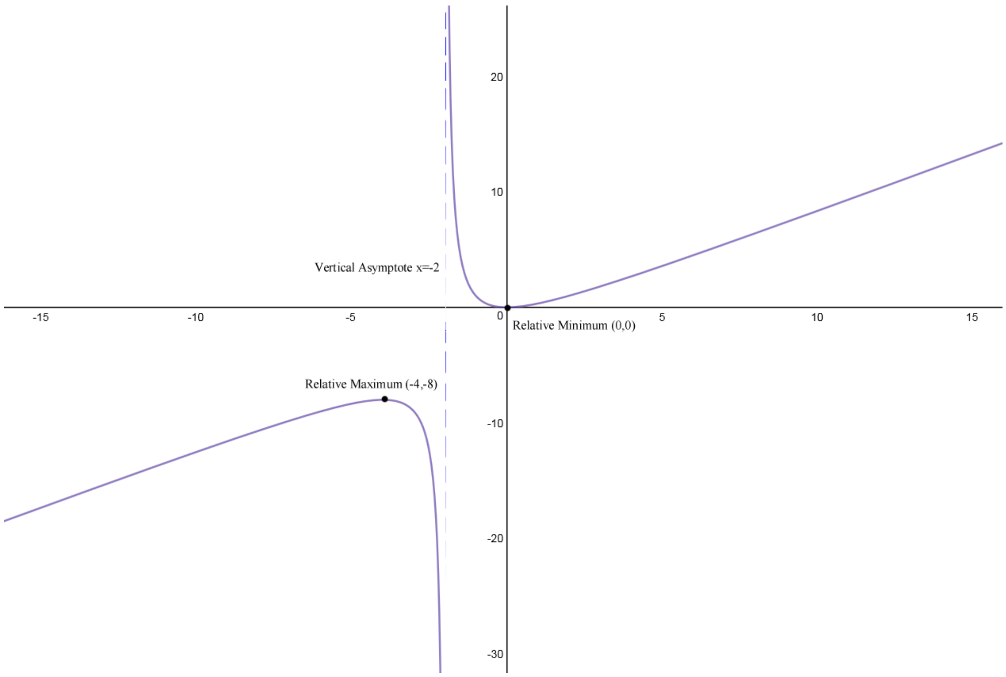
15. Domain $\mathbb{R} = (-\infty, \infty)$; y-int$= (0, 93)$; Increasing on: $(-\infty, 0) \cup (4, \infty)$; Decreasing on: $(0, 4)$; Relative maximum: $f(0) = 93$; Relative minimum: $f(4) = -163$; Concave upward on: $(3, \infty)$; Concave downward on: $(-\infty, 3)$; Inflection point: $(3, -69)$.



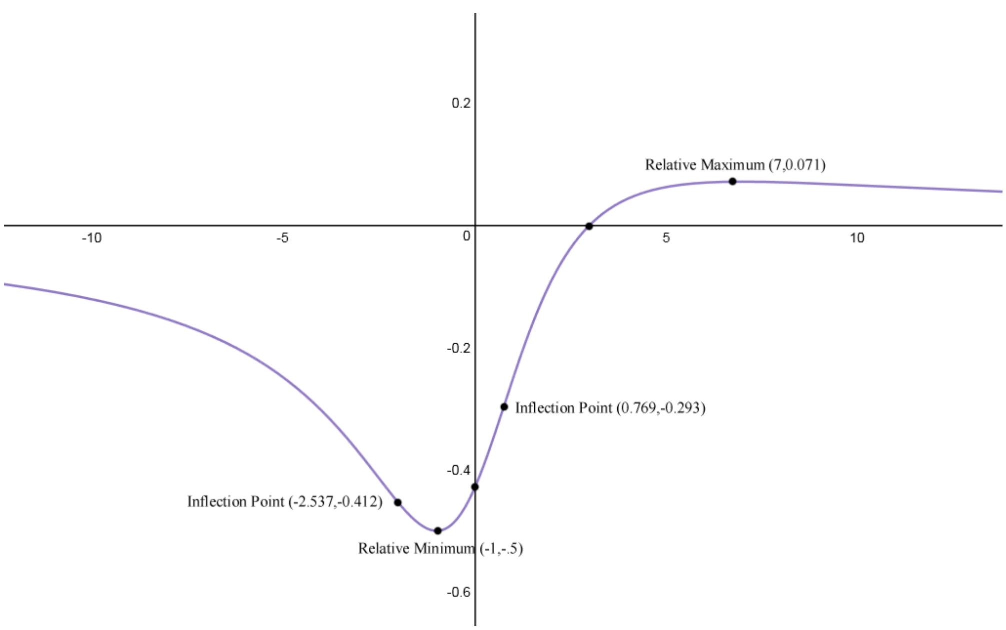
16. Domain $\mathbb{R} = (-\infty, \infty)$; y-int$= 0$; x-int$= 0, 1/3$; Increasing on: $(-\infty, 1/4)$; Decreasing on: $(1/4, \infty)$; Relative maximum: $f(1/4) = 0.004$; Concave upward on: $(0, 1/6)$; Concave downward on: $(-\infty, 0) \cup (1/6, \infty)$; Inflection point: $(0, 0)$ and $(1/6, 0.0021)$.



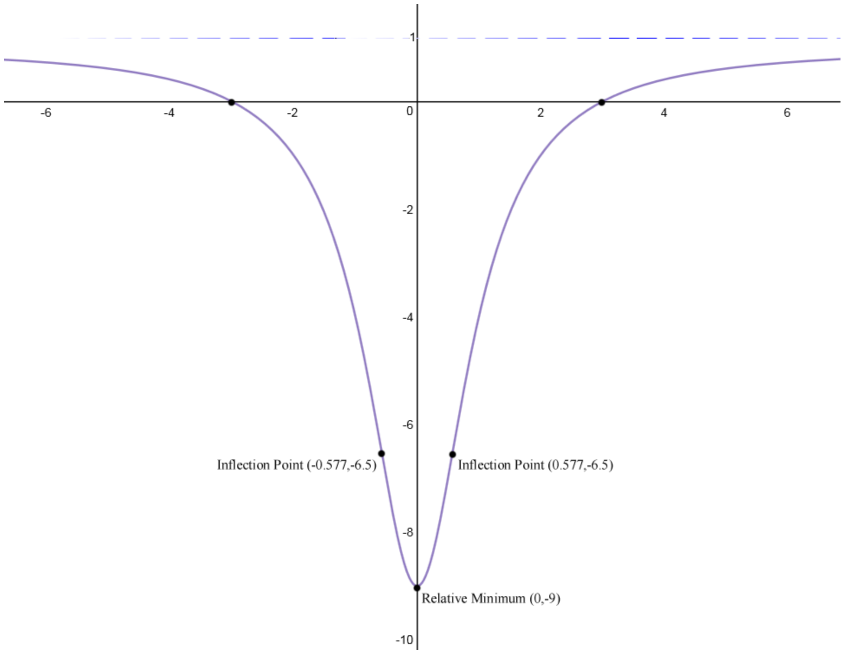
17. Domain $\mathbb{R}\backslash\{-2\}$; y-int$=0$; x-int$=0$ Vertical asymptote $x=-2$; Increasing on: $(-\infty,-4)\cup(0,\infty)$; Decreasing on: $(-4,0)$; Relative maximum: $f(-4)=-8$; Relative minimum: $f(0)=0$; Concave upward on: $(-2,\infty)$; Concave downward on: $(\infty,-2)$; No inflection points.



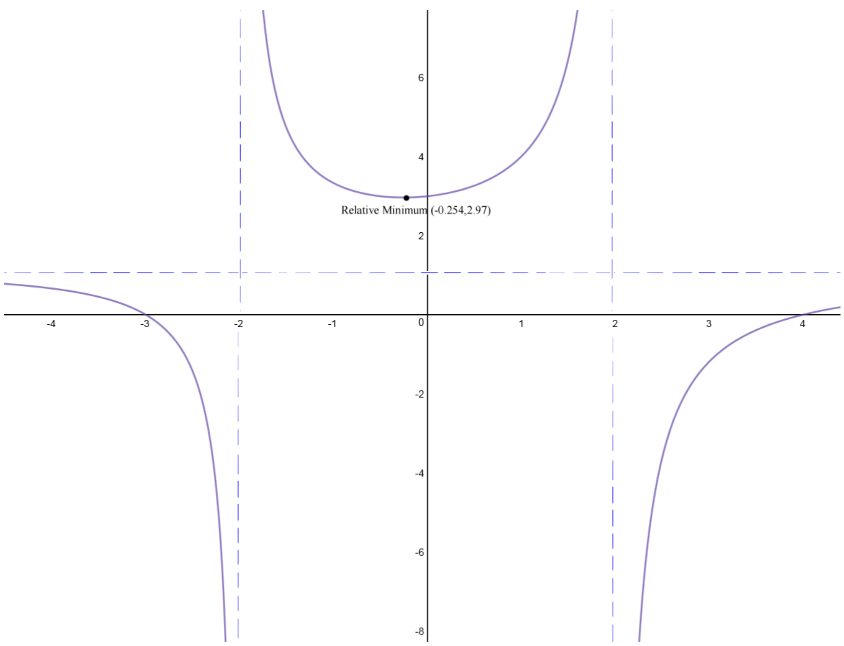
18. Domain $\mathbb{R}$; y-int$=-3/7$; x-int$=3$ Horizontal asymptote $y=0$; Increasing on: $(-1,7)$; Decreasing on: $(-\infty,-1)\cup(7,\infty)$; Relative maximum: $f(7)=0.071$; Relative minimum: $f(-1)=-0.5$; Concave upward on: $(-2.537,0.769)$; Concave downward on: $(-\infty,-2.537)\cup(0.769,\infty)$; Inflection points: $(-2.537,-0.412)$ and $(0.769,-0.293)$. Note: Precise locations require using equation solver tool.



19. Domain $\mathbb{R}$; y-int$= -9$; x-int$= \pm 3$; Horizontal asymptote $y = 1$; Increasing on: $(0, \infty)$; Decreasing on: $(-\infty, 0)$; Relative minimum: $f(0) = -9$; Concave upward on: $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; Concave downward on: $\left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$; Inflection points: $\left(-\frac{1}{\sqrt{3}}, -6.5\right)$ and $\left(\frac{1}{\sqrt{3}}, -6.5\right)$.



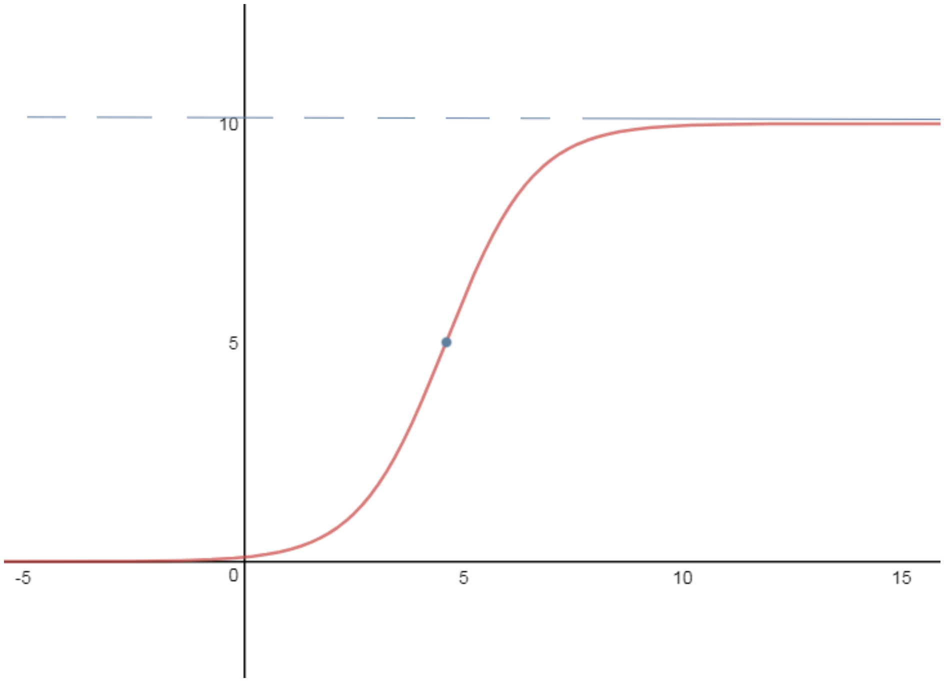
20. Domain $\mathbb{R} \backslash (\{-2\} \cup \{2\})$; y-int$= 3$; x-int$= -3, 4$; Vertical asymptote $x = -2$ and $x = 2$; Horizontal asymptote $y = 1$; Increasing on: $(0, \infty)$; Decreasing on: $(-\infty, 0)$; Relative minimum: $f(-0.254, 2.97)$; Concave upward on: $(-2, 2)$; Concave downward on: $(-\infty, -2) \cup (2, \infty)$; No inflection point.



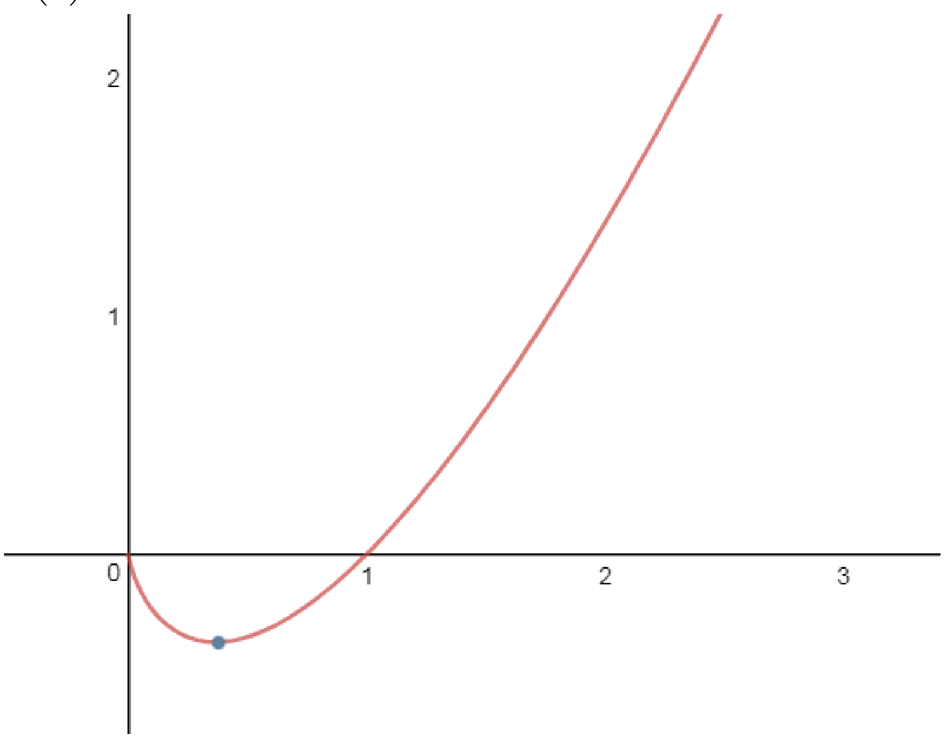
8 Exercises (page 19)

1. $f'(x) = e^{4x^{23}-x^4}(92x^{22} - 4x^3)$
2. $f'(x) = e^{2x} + 2xe^{2x}$
3. $f'(x) = \dfrac{-xe^{-x}}{(x+1)^2}$
4. $f'(x) = 4(x^3 + e^x + 3)^3(3x^2 + e^x)$
5. $f'(x) = \dfrac{4x}{x^2 - 4}$
6. $f'(x) = \dfrac{1}{x}$
7. $f'(x) = \ln(x) + 1$
8. $f'(x) = \dfrac{3(\ln(x) - 5)^2}{x}$
9. $f'(x) = \dfrac{(2x-1)\ln(x) - (x-1)}{\ln(x)^2}$
10. $f'(x) = \ln(x) + 1 - e^{2x}(2x^2 - 1)$
11. $\dfrac{dy}{dx} = x^{3x}(3x\ln(x) + 3x + 1)$
12. $\dfrac{dy}{dx} = (2x^2 - 4)^{\frac{1}{x}}\left(\dfrac{4}{x^2 - 2} - \dfrac{\ln(2x^2 - 4)}{x^2}\right)$
13. (a) $w(t) = 50(3^t)$
    (b) $w(6) = 36,450$ (yuck!)
    (c) $w'(0) = 54.9$ worms per week
    (d) $w'(6) = 40,044$ worms per week

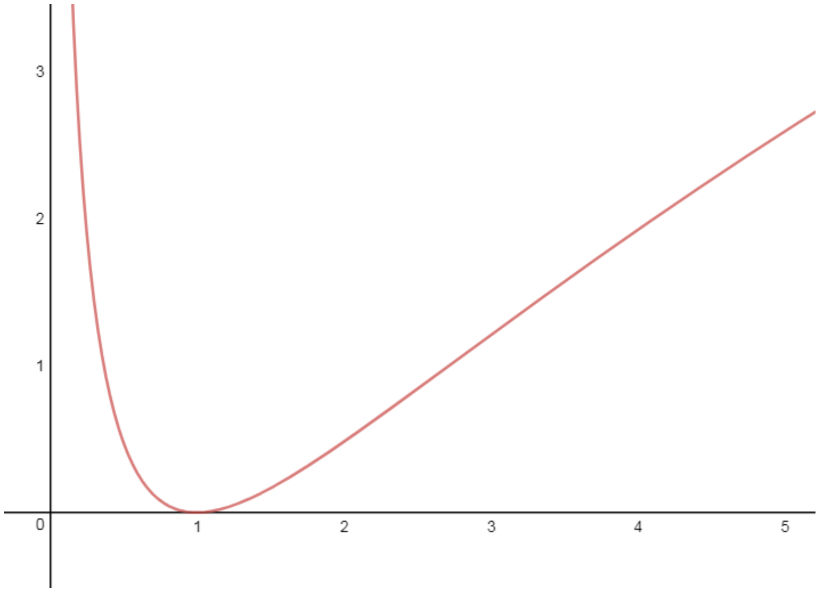
14. Domain $\mathbb{R}$; y-int$= 10/101$; Horizontal asymptotes $y = 0$ and $y = 10$; Increasing on: $(-\infty, \infty)$; Concave upward on: $(-\infty, \ln 100)$; Concave downward on: $(\ln 100, \infty)$; Inflection point: $(\ln 100, 5)$.



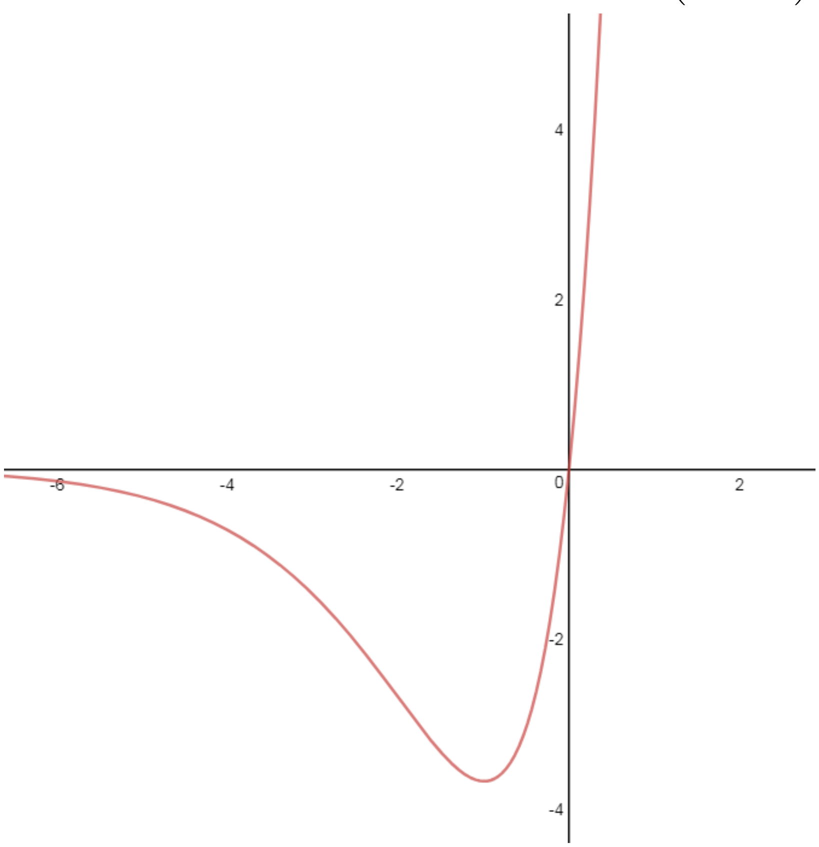
15. Domain $(0, \infty)$; x-int$= 1$; Increasing on: $\left(\frac{1}{e}, \infty\right)$; Decreasing on: $\left(0, \frac{1}{e}\right)$; Relative minimum: $f\left(\frac{1}{e}\right) = -\frac{1}{e}$; Concave upward on: $(0, \infty)$; No inflection points.



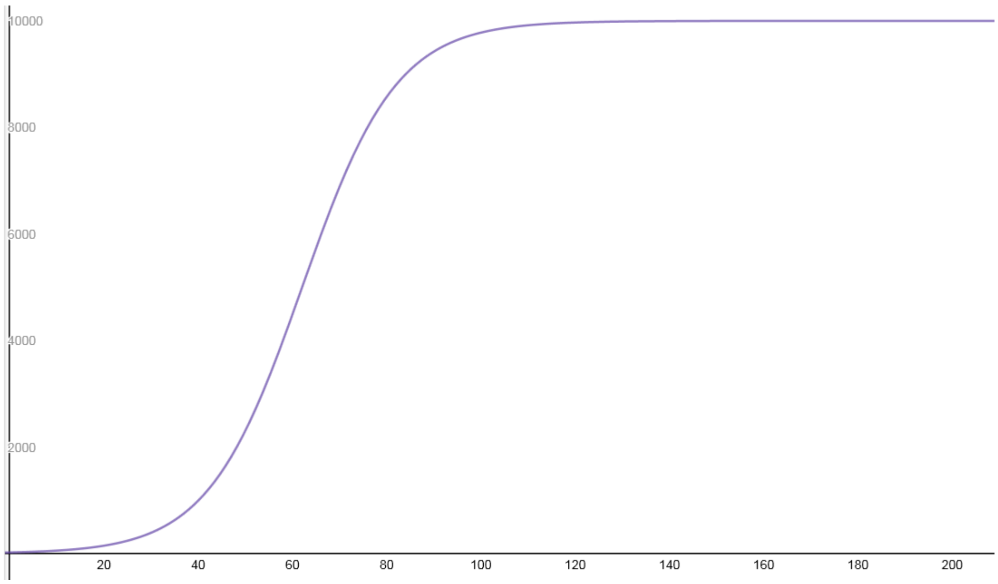
16. Domain $\mathbb{R}$; x-int$=1$; Increasing on: $(1,\infty)$; Decreasing on: $(0,1)$; Relative minimum: $f(1)=0$; Concave upward on: $(0,e)$; Concave downward on: (e,∞); Inflection point: $(e,1)$.



17. Domain $\mathbb{R}$; y-int$=0$; x-int$=0$; Horizontal asymptote $y=0$; Increasing on: $(-1,\infty)$; Decreasing on: $(-\infty,-1)$; Relative minimum: $f(-1)=-10/e$; Concave upward on: $(-2,\infty)$; Concave downward on: $(-\infty,-2)$; Inflection point: $\left(-2,-\dfrac{20}{e^2}\right)$.



18. max of -0.847 at $x = .707$.
19. max of 0.54 at $x = -2$ and min of 0 at $x = 0$.
20. Minimum of \$4.08 at $t = 20$.
21. Maximum effectiveness at $t = 10$ hours.
22. (a) 20
    (b) This is the limit at infinity, so $10,000$.
    (c) 48.3 hours
    (d) $t = 62$
    (e)



9 Exercises (page 21)

1. $F(x) = x^3 - \frac{5}{2}x^2 + 6x + C$
2. $F(x) = \frac{1}{2}x^2 - \frac{4}{x} + C$
3. $F(x) = \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + C$
4. $F(x) = \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{7}x^7 + e^2x + C$
5. $F(x) = 3\ln(x) + C$
6. $F(x) = \frac{1}{2}e^{2x} + C$
7. $\frac{4x^3}{3} + 10\sqrt{x} - 4x + C$
8. $\frac{x^3}{3} - \frac{x^2}{2} - 12x + C$

9. $\dfrac{2\sqrt{2}x^{5/2}}{5} + C$

10. $\dfrac{x^2}{2} + \dfrac{2\sqrt{2}x^{3/2}}{3} + C$

11. 18

12. -3

13. $(e^{10} - \dfrac{100}{2}) - (e^{-1} - \dfrac{(-1)^2}{2}) = e^{10} - \dfrac{1}{e} - 50.5$

14. $\ln(e^2) - \ln(1) = 2 - 0 = 2$

15. $-\dfrac{1}{12}\left(x^3 + 3x^2 + 4\right)^{-4} + C$

16. $\dfrac{1}{3}\left(5x^2 + 2x\right)^{\frac{3}{2}} + C$

17. $\dfrac{1}{40}(4x+1)^{\frac{5}{2}} - \dfrac{1}{24}(4x+1)^{\frac{3}{2}} + C$

18. $e^{x^2-4} + C$

19. $\ln(2x^2 - 4x) + C$

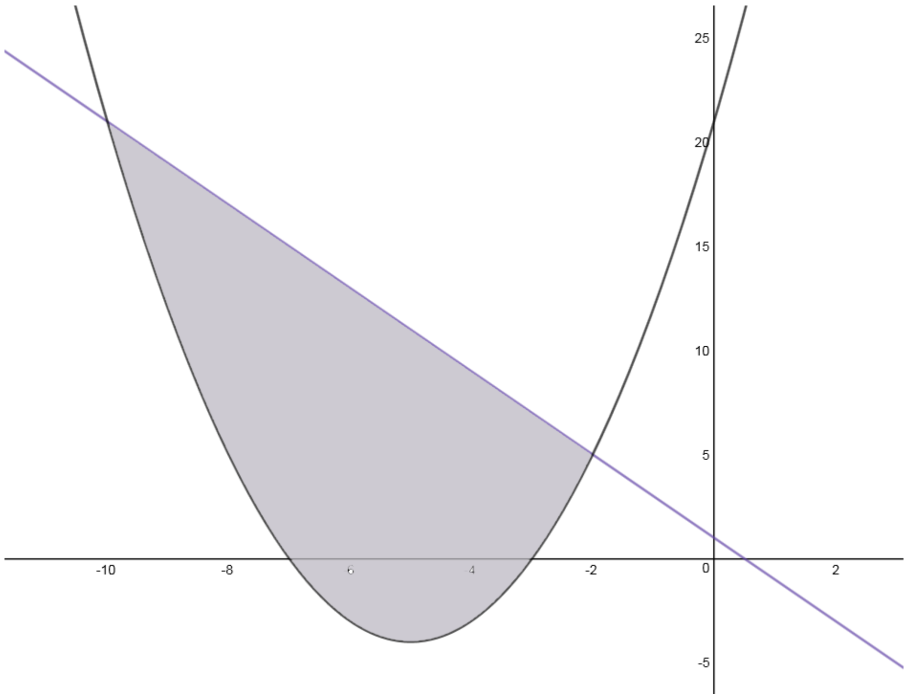
20. Using $u = x^4 + 9$, integral is $\displaystyle\int_9^{25} u^{\frac{1}{2}} \frac{du}{4} = \frac{49}{3}$

21. Using $u = x^2$, integral is $= \displaystyle\int_0^{25} e^u \frac{du}{2} \frac{1}{2}(e^{25} - 1)$

22. Using $u = 2x^2 + 1$, integral is $\displaystyle\int_3^{19} u^{-2} \frac{du}{4} = \frac{4}{57}$

23. $\dfrac{10\sqrt{5}}{3}$

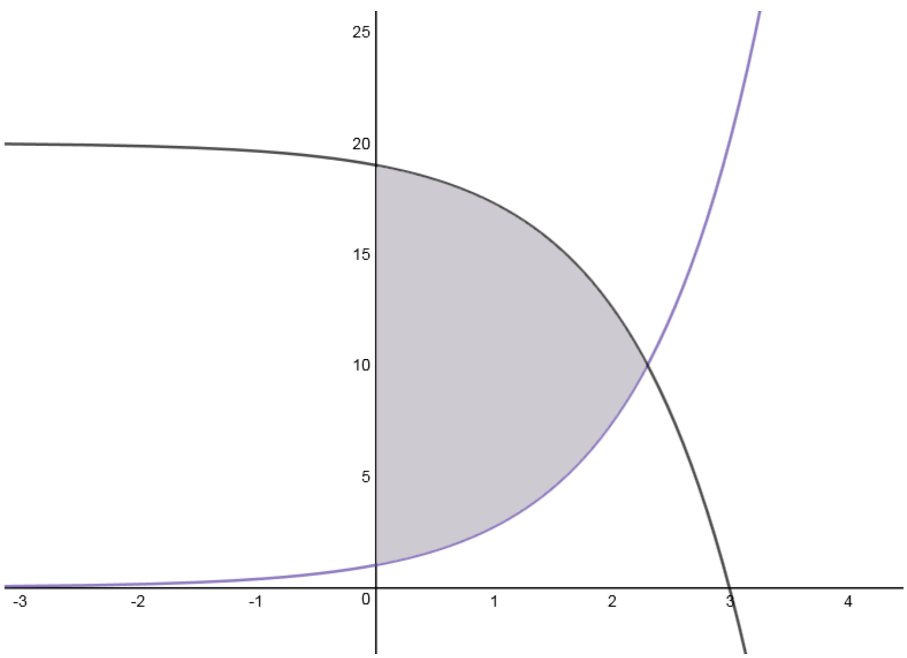
24. (a)



(b) They intersect at $x = -2$ and $x = -10$

(c) $\dfrac{256}{3}$

25. (a)



(b) They intersect at $x = \ln(10) = 2.3$

(c) 28

26. $\dfrac{159}{4}$

27. 39

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Version 1.3, 3 November 2008

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