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PUBLIC PENSION SCHEMES  
IN OVERLAPPING GENERATIONS ECONOMIES

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## Public Pension Schemes in Overlapping Generations Economies

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### Abstract

As many nations experience population aging, due to both declines in fertility and rising life expectancies, policymakers are grappling with the reform of public pension systems. Two general classes of public pension systems are examined in an overlapping generations general equilibrium context, the “pay-as-you-go” (PAYGO) system, and the universal fully funded pension (UFFP) system. Simulations show that variations in population structures resulting from changes in population growth rates have significant effects on the pension program in PAYGO model economies, but no effect on the pension program in UFFP model economies. In addition, changes in real interest rates have significant effects on the pension program in UFFP model economies, but no impact on the pension program in PAYGO economies.

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## I. Introduction

Many of the industrialized nations are struggling with public pension reform. The combination of increasing life expectancies and lower population growth rates has led to increases in future dependency rates and pension obligations for public pay-as-you-go (PAYGO) plans. Some analysts have proposed the privatization of public plans following the Chilean model, whereby the government guarantees a minimum benefit regardless of investment fund performance. However, this plan, like other privately administered plans, suffers from very high administrative costs. Others have pointed to the plans in Malaysia and Singapore that are privately funded but publicly administered. These programs, along with the Chilean program, do not have the advantage of pooling longevity risks. Abel (1986) suggested a “Fully Funded Social Security” program with compulsory participation that would benefit from low public administration costs and longevity risk pooling. However, Abel did not address the design of such a program. Ho (1997) made the first proposal for the design of such a program, termed a universal fully funded pension, UFFP. This paper extends Ho’s proposal by presenting a version of a UFFP fund, and comparing it to a PAYGO program with similar payouts to beneficiaries. This paper also presents an overlapping generations general equilibrium model to compare welfare of agents in model economies with different population age structures, in both PAYGO and UFFP environments.

The typical pay-as-you-go pension programs are funded out of general tax revenue, even though contributors pay premiums on their earnings to fund the program. With an increase in the dependency ratio of a population, the pension liability rises, resulting in either (i) an increase in contribution rates, (ii) a decrease in benefit rates, (iii) a postponement of the retirement age, (iv) an increase in other tax revenues, or (v) some combination of these four items. This raises intergenerational equity issues as some generations benefit at the cost of others in terms of the returns on their contributions into the pension program. Only a fully-funded pension program is capable of avoiding these intergenerational concerns.

Private pensions are fully-funded, however, they require that each individual fully funds his or her own pension in the face of longevity risk. The problem that the individual faces is that of fully anticipating his or her own lifetime. If the projected lifetime is shorter than the actual lifetime, the individual risks poverty and will be forced to rely on family, charity, or other social programs. If the projected lifetime is longer than the actual lifetime, the individual leaves an

unexpected bequest, resulting in a suboptimal outcome. Private programs also suffer from high administrative costs.

To overcome the problems of both pay-as-you-go and private pension programs, Abel (1987) proposed a public pension system that is fully funded, but does not require each individual to fully fund his or her own account. Rather, the proposed program is one where each cohort or generation fully funds their aggregate account. In effect, each cohort pools their longevity risk. The program is also publicly administered and does not suffer from high administration costs.

Ho (1987) elaborated on Abel's proposal, introducing a universal fully funded pension program, UFFP. Each member of a cohort is required to contribute an equal share into the fund, and all surviving members will receive an equal benefit during the retirement years. The public pension program is setup so that members of each cohort receive a base minimum of funding to support basic consumption needs during the retirement phase. Members will be free to privately save in order to draw additional retirement funds. Unlike at the individual level, there is little uncertainty regarding life expectancy, and so it is easier to specify jointly the contribution and benefit-payout levels for the program. The program is mandatory to eliminate moral hazard and adverse selection – all members of the cohort must participate. However, the program can take into account members of society who cannot make contributions, but should receive public funding and support during the retirement years.

This paper considers Ho's proposal to evaluate the effects of such a program on welfare in comparison to the pay-as-you-go public pension program. The study uses overlapping generations general equilibrium model economies similar to those used by Huggett (1996), Rios-Rull (2001), and Wilson (2001, 2003). The model economies exhibit different population age structures in order to investigate the welfare issues facing model economies with the different public pension systems. The modeling environment is flexible enough to incorporate heterogeneity in employment status across individuals and over time (including bouts of unemployment), longevity risk, and both private and public pension systems. The modeling results include the calculation of expected lifetime utility measures for agents in the two different pension programs, given the different population age structures of the model economies. Savings rates will also be calculated for the different model economies.

## II. Public Pension Systems

The majority of national public pension or social security programs were designed under the “pay-as-you-go” model, whereby current benefits to retirees are paid by payroll taxes on labour earnings by current workers. The pension program is typically designed so that current contributions equal current benefits, so that the pension does not accumulate a surplus or deficit. Any surplus or deficit is absorbed by the government’s general revenue fund. If the pension fund begins to accumulate a surplus, the PAYGO nature of the plan would suggest that payroll tax rates should be reduced and offer tax relief to workers, all else equal. If, however, the pension fund accumulates a deficit, the PAYGO nature of the plan suggests that the payroll tax rate needs to be raised, thereby increasing the tax burden of workers, all else equal.<sup>1</sup>

As a simple illustrative example, consider an environment where agents live for two periods, work only when young and receive pension benefits when old. A government exists only to collect pension contributions and distribute them amongst retirees. Let  $N_{1t}$  equal the proportion of young workers in an economy, and let  $N_{2t}$  be the proportion of retirees. Assuming that workers earn wage  $w_{1t}$ , and retirees receive a benefit  $b_{2t}$ , then the payroll tax rate,  $p_{rt}$  in the PAYGO plan is calculated as:

$$p_{rt} = \frac{N_{2t}b_{2t}}{N_{1t}w_{1t}} \quad (\text{A})$$

In this very simple case, if the proportion of retirees rises, all else equal, then the pension payroll tax rate must rise. This is the problem now facing many public PAYGO programs, as the proportion of retirees relative to workers is increasing because of increased longevity and declining fertility rates and labour force growth, so that there are pressures to increase the pension payroll tax rate to maintain the sustainability of the PAYGO program. The Canadian Government, facing population aging, raised the pension payroll tax rate from 3.6% of taxable earnings in 1986, to 9.9% in 2003 (Finance Canada, 1999). Of course there are other proposals to sustain the PAYGO system, including an increase in the retirement age which has the effect of lowering the ratio of retirees to workers  $N_{2t}/N_{1t}$ , and/or a decrease in the level of benefits paid to retirees,  $b_{2t}$ , to maintain a low pension payroll tax rate.

In contrast to the PAYGO pension system, Ho (1997) presented a Universal Fully Funded Pension (UFFP) system, whereby each cohort or generation pays into a fund when working, from

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<sup>1</sup> Other adjustments may be made, including changes to the age of retirement, and the level of retirement benefits.

which the same cohort draws when retired. The very simple illustrative example above is adjusted for the UFFP system so that the pension program is balanced for each cohort, rather than balanced at any one period in time. The pension payroll tax rate is now calculated as follows:

$$p_{rt} = E_t \left[ \frac{N_{2t+1} b_{2t+1}}{N_{1t} w_{1t} (1+r_t)} \right] \quad (B)$$

Now, the pension payroll rate depends entirely on the environment facing the individual cohort, namely the survival rate of workers that reach retirement age,  $N_{2t+1}/N_{1t}$  (as well as the interest rate, wage rate and benefit level).

Extending this very simple example to the multi-period case does not involve any adjustment to the PAYGO premium rate calculation. However, the the UFFP program must be balanced for each cohort or generation so that the present value of contributions during the working years is equal to the present value of benefits. This budget constraint becomes the following for the cohort just starting work at age  $I_w$ , at time  $t$ , that will retire at age  $I_r$ , and whose cohort size is reduced each period due to death (note that  $N_{i,t} \neq N_{i+1,t+1}$ ), with a maximum lifetime of  $I$ :

$$E_t \left[ \sum_{i=I_w}^{I_r-1} N_{i,t+i-I_w} p_{t+i-I_w} w_{i,t+i-I_w} (1+r_{t+i-I_w})^{I_w-i} - \sum_{i=I_r}^I N_{i,t+i-I_w} b_{t+i-I_w} (1+r_{t+i-I_w})^{I_w-i} \right] = 0 \quad (C)$$

How do the two different plans respond to demographic changes? The major demographic disturbances are due to changes in longevity and survival rates by age, and due to changes in fertility or labour force growth. First, any unexpected increase in longevity and survival rates of individuals in the economy with a UFFP plan will affect the cohort experiencing this change only, by raising the number of survivors of the cohort at higher age categories. In the simple two-period case, the ratio  $N_{2t+1}/N_{1t}$  increases, and results in a decrease in benefit levels payable by the pension fund. If this type of change occurs in the PAYGO environment, then this change can affect the current workers via an increase in the pension payroll tax rate, to pay for the increased level of aggregate pension benefits. An alternative for the PAYGO plan is to reduce the benefit levels of retirees or delay the age of retirement so that the working generation is not adversely affected. Policymakers in an economy with a PAYGO plan, however, may choose to subsidize beneficiaries by raising contribution rates of young workers if the elderly have more political power than the young. Policymakers in an economy with a UFFP plan may be better able to avoid a political movement of beneficiaries hoping to get a government subsidy,

paid by young workers, to offset any loss in expected benefits. Any change in fertility rates and labour force growth will not affect pension benefit levels and payroll tax rates in the UFFP plan, but under the PAYGO plan, these changes will affect the worker-to-retiree ratio and may force changes to contribution rates. Policymakers in the PAYGO environment will struggle with the decision as to which generation should shoulder the effects of the demographic change, whereas policymakers in the UFFP environment may face less political backlash and the effect of demographic change is borne by the generation experiencing the change itself.

At this point, it would be best to note two key items in the Ho (1997) proposal and how they will be addressed in this paper. First, Ho proposed that all members of a given cohort will contribute to and draw from the same pension fund. Ho realized that agents may not have the means to contribute to the plan due to changes in employment status. In the modeling setup that follows, agents who are unemployed will not make contributions to the pension plan. The alternative to this would be to have unemployment benefits taxed back to make contributions to the program, reducing the net amount of unemployment benefits received by beneficiaries. Those who are not in the labour force also would draw from the pension fund when past the retirement age, but may not have the personal means to contribute to the fund when of working age. In this paper, a subset of the population will never be labour force participants, and those who do participate will do so for their working-age lives. Those who do not participate in the labour force will receive welfare payments from a general fund while of working age, then will draw from the UFFP fund while elderly. Those on welfare do not make contributions to the fund.<sup>2</sup> Second, Ho proposed that contributions would be set at a fixed level each period. Contributions will be set at a fixed proportion of wage earnings in this paper, corresponding to current payroll taxation methods for some PAYGO pension programs, including the Canada Pension Plan.

A very simple illustration has been provided to examine the implications of demographic effects on the two different public pension plans. The following section is an attempt to provide a richer environment in which to examine the differences between the two public pension systems. The environment will not be complicated and realistic enough to answer many of the questions policymakers ask, but will provide answers to a few questions. How do payroll tax rates differ between pension programs under different population distributions if benefit levels are equivalent across the two plans? How do changes in interest rates affect the pension plans? How are savings rates and foreign capital dependence influenced by the type of public pension plan in

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<sup>2</sup> Alternatively, welfare payments could be taxed to provide contributions to the fund.

the economy? How do labour force participation rates affect the economies under different public pension plan regimes?

### III. Overlapping Generations Life-Cycle Economies

Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), Hubbard, Skinner and Zeldes (1995), Bohn (1999), Huggett and Ventura (1999), and Emery and Rongve (1999), among others, have used the overlapping generations framework to examine social security, saving and capital accumulation, and public pension reform in differing population environments. It has been found that in model economies, public pensions can have a considerable impact on welfare and distort savings incentives. The overlapping generations framework is rich enough to incorporate uncertain finite lifetimes, heterogeneity of various types including age and employment history, and other changing characteristics over the life cycle. In this study, the overlapping generations framework will be used to examine the differences between the pay-as-you-go (PAYGO) pension system and the universal, fully-funded pension (UFFP) system.

#### **The Environment**

The modeling environment is similar to that used by Huggett (1996), and Wilson (2001). The model economies to be examined are populated by overlapping generations of rational, forward-looking agents. Agents are heterogeneous in age and employment history. Agents live to a maximum of  $I$  periods, start work at  $I_w$ , and retire at  $I_r$ . The model takes as given that agents retire at age  $I_r$ , whether a pension program exists or not.<sup>3</sup>

The population distribution by age cohort at a point in time is described by the vector  $\mu_t$ , where  $\mu_t = [\mu_1 \mu_2 \dots \mu_t]^\top$ . The age  $i$  cohort consists of  $\mu_i$  agents. Agents face uncertain lifetimes, but know age-dependent survival probabilities. The probability of surviving from age  $i$  to age  $i+1$  is  $s_i$ . The unconditional probability of surviving to age  $k$  is  $s(k) = \prod_{j=1}^{k-1} s_j$ . There is no migration in the model economies, so that  $\mu_{i+1,t+1} = s_i \mu_{i,t}$ . In a steady-state, age  $i$  agents will make up a constant fraction of the population at any point in time, and the population growth rate will be  $n$ . In the steady state,  $\mu_{i,t} = (1+n)\mu_{i,t-1}$ . A fixed proportion of individuals,  $\pi^l$ , in each age cohort will be labour force participants for the duration of their working lives. All other individuals

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<sup>3</sup> The Canada Pension Plan was introduced in 1966, and most of the decline in the labour force participation of men over 65 years of age had already occurred. The labour force participation rate of males aged 65 and older fell steadily from 30% in 1960 to 20% in 1970, and has fluctuated around 10% in the last decade. The induced retirement effect of pensions is explored in Feldstein (1974), and will be left to be examined in future work in the modeling framework described herein.

(proportion  $1 - \pi^l$  of the population, and of each age cohort) will not participate in the labour force at any time during their lifetimes. These individuals will receive welfare benefits from the government.

At the aggregate level, there is no uncertainty. Production is deterministic and is governed by a constant returns to scale technology of the following form:

$$Y_t = F(K_t, L_t) = BK_t^\alpha (A_t L_t)^{1-\alpha}. \quad (1)$$

Labour-augmenting technological progress,  $A_t$ , grows at a constant rate  $g$ .  $B$  is a scale parameter. Capital depreciates at a constant rate,  $\delta$ . Model economies will be investigated under the small open economy framework, with varying real interest rates, assumed set on international markets.

Agents maximize expected utility over a standard composite good, with time-separable preferences and discount factor,  $\beta$ . The composite good can be used as capital, and earns a risk-free one period return per unit,  $R$ . Agents maximize expected lifetime utility, given by:

$$\max E_{t_w} \left( U(c_{t_w}) + \sum_{j=t_w}^I \beta^j s(j-1) U(c_j) \right), \quad \text{where } U(c) = \frac{c^{1-\sigma}}{1-\sigma}. \quad (2)$$

Agents face a positive probability of death at all ages. Correspondingly, agents may die with assets, and leave bequests to surviving agents. These “accidental” bequests have been treated several different ways in similar simulation exercises. Rios-Rull (2001) introduced a market whereby agents write contracts with members of their own age cohort to share wealth or debts of those members who die before the terminal age,  $I$ . In the model economies examined by Huggett (1996), accidental bequests are fully taxed and redistributed amongst all surviving agents in the economy. Storesletten (2000) had accidental bequests fully transferred to newborns. The model economies examined in this study will have accidental bequests distributed equally amongst the adult population (those of age  $I_w$  or older), as inheritances tend in practice to be distributed amongst family members of varying ages.

Agents in the labour force are endowed with labour efficiency units that are dependent on age, denoted by  $e_i$ . Labour force participants face the probability  $\pi_i^e$  of being employed, and  $\pi_i^u = 1 - \pi_i^e$  of being unemployed in any given period. The probability of being employed in a given period for an individual is independent of being employed in any other time period. If employed,  $l(h_i) = 1$ . If unemployed,  $l(h_i) = 0$ . In these economies, while there is no aggregate uncertainty, there exists uncertainty at the individual level.

The government provides retirement benefits and employment insurance to agents in the labour force. Individuals who survive past the age of  $I_{r-1}$  receive retirement benefits at a rate

equal to a level  $b_{i,t}e_r w$ , indicative of the employment history of the average individual in that cohort ( $e_i = e_r$  for  $i \geq I_r$ ).<sup>4</sup> Unemployed agents receive a benefit equal to a proportion of their expected earnings if employed,  $b_{i,t}e_i$ . The government finances the employment insurance program by levying a payroll tax equal to a proportion,  $p_e$ , on labour earnings, and finances the pension program with payroll tax equal to  $p_r$  on labour earnings. The government also levies a flat income tax of rate  $\tau$ , levied equally on wage earnings, capital earnings, pension or employment insurance benefits, and transfers of accidental bequests, to pay for government consumption.

In period  $t$ , labour force participants of age  $i$  choose consumption,  $c_{i,t}$ , and asset holdings,  $a'_{i,t} = a_{i+1,t+1}$  that are carried into the next period, with the following constraints:

$$c_{i,t} + a'_{i,t} \leq W_t = a_{i,t} + (r_t a_{i,t} + (1 - p_e - p_r) e_i w_t l(h_i) + b_{i,t} e_i w_t (1 - l(h_i)) + T_t) \times (1 - \tau_t) \quad \text{and}$$

$$c_{i,t} \geq 0, \quad \text{and} \quad a'_{i,t} \geq 0 \quad \text{for all } i, \text{ and } t. \quad (3)$$

In this context, the value of  $T_t$  is determined by the amount of accidental bequests left by those who die in period  $t$ , before the age of  $I$ . Agents are liquidity constrained and cannot hold negative net assets. Agents are not capable of borrowing without collateral greater than or equal to the amount borrowed, but are allowed to mortgage capital. In this case agents are unable to borrow against the promise of future income to finance consumption, and cannot hold negative net asset levels. In the life-cycle model economy with survival uncertainty, this assumption restricts agents' borrowing so that they do not accumulate debts and die before repayment.

### **Government Programs**

The government levies a payroll tax equal to a rate  $p_e$  of employment earnings to finance the employment insurance program, and a payroll tax rate of  $p_r$  to finance the pension program. The government also levies an income tax to finance government consumption including a welfare program to provide income to non-working individuals in the economy. Welfare recipients do not accumulate wealth or debt. Model economies will be examined with different methods of financing government programs. In the first case, model economies will have an employment insurance program and a public pension system that can be characterized as pay-as-you-go programs. The social welfare program will be financed out of general tax revenues (also on the pay-as-you-go model). In the second case, the model economies will have a pay-as-you-go

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<sup>4</sup> This is done for simplicity, instead of keeping track of each individual's employment history and having benefits dependent on an individual's employment history.

employment insurance program for those of working age, and a universal fully-funded pension program that covers all those aged 65 years or more. Since the UFFP covers all individuals including those who did not work before the age of retirement, the social welfare program will cover only individuals of working age, and will be finance by general tax revenues.

The government pay-as-you go employment insurance and pension program budgets must be balanced on an annual basis. Total revenues from the payroll tax will be equal to the expenditures in providing employment insurance benefits to the unemployed and pensions to those who are retired. The government budget constraint for the employment insurance program is:

$$\sum_{i=I_w}^{i=I_r-1} \pi_i^e (\pi^l \mu_i) p_e e_i w = \sum_{i=I_w}^{i=I_r-1} \pi_i^u (\pi^l \mu_i) b_i e_i w. \quad (4)$$

The government budget constraint for the pay-as-you-go public pension program is:

$$\sum_{i=I_w}^{i=I_r-1} \pi_i^e (\pi^l \mu_i) p_r e_i w = \sum_{i=I_r}^{i=I} (\pi^l \mu_i) b_i e_i w. \quad (5)$$

Note that  $(\pi^l \mu_i)$  equals the amount of labour force participants of age cohort  $i$ . The unemployed do not pay pension or employment insurance premia.

In model economies with a UFFP program, the government budget constraint for the employment insurance program is unchanged, but the budget for the public pension system must now be balanced for each cohort over the lifecycle, rather than balanced each year. The constraint for the cohort that begins work at time  $t+I_w$  becomes:

$$PV \left[ \sum_{i=I_w}^{i=I_r-1} \pi_{i,t+i}^e (\pi^l \mu_{i,t+i}) p_{e,t+i} e_{i,t+i} w_t \right] = PV \left[ \sum_{i=I_r}^{i=I} \mu_{i,t+i} b_{i,t+i} e_{i,t+i} w_t \right] \quad (6)$$

The overall government budget must also be balanced. In this case, it is assumed that the government uses all income tax revenues for government spending including the welfare program. Government expenditures do not enter into the individual utility function.

### **Steady-State Equilibrium**

In these model economies, population size will not remain constant, but the distribution of the population across age cohorts will remain constant. There will be growth in the total population, and of each group of a given age, at annual rate  $n$ , which may be positive or negative. In addition, there will be labour-augmenting technological progress, with an annual growth rate of  $g$ . All variables expressed as rates are fixed over time. The variables for capital and labour inputs, wages, benefit levels and the transfers of accidental bequests grow at constant rates over time.

The treatment of pension and welfare benefits in a steady state economy with technological progress becomes complicated. If benefits are fixed in real terms, then over time, the welfare and pension programs will become irrelevant, in terms of funding resources, since their share of total income will steadily decline, and contribution rates will fall to almost nil. Income inequality between labour force participants and those who do not participate will rise dramatically over time. To avoid this complication, welfare and pension benefits will be set at a fixed proportion of wage rates, and will be allowed to grow over time at the same rate as labour earnings. Income inequality between haves and have-nots will not change over time.

The method of computing the steady state is similar to the method described in Wilson (2001). First, the total effective labour supply in the economy is calculated. Then given the age distribution of the population, age-specific unemployment rates, and pension benefit rates, the expected total amount of pension and employment insurance benefits is calculated. The premium rates for both employment insurance and public pensions are then calculated. A guess of the transfer of bequests per adult,  $T$ , is made, and the level of optimal asset holdings of agents of all ages in the economy can be computed recursively, given this level of transfers. Once the level of optimal asset holdings is calculated, then representative sample populations are generated, agents' asset holdings are calculated, and the value of accidental bequests (along with other variables) is calculated and compared to the guessed level of transfers. If the two values are equal the procedure is completed, otherwise, the guessed level of transfers is updated and the computational procedure is repeated until convergence is achieved.<sup>5</sup>

In the steady state, the budget constraints for the employment and PAYGO pension programs require no adjustment. However, the budget constraint for the UFFP program becomes:

$$\sum_{i=I_w}^{i=I_r-1} \pi_i^e (\mu^l \mu_i) (1+n)^{i-I_r} p_r e_i w (1+g)^{i-I_r} (1+r)^{I_r-i} = \sum_{i=I_r}^{i=I} \mu_i (1+n)^{I_r-i} b_i e_i w (1+g)^{I_r-i} (1+r)^{i-I_r} \quad (7)$$

This is because age cohorts grow at the population growth rate over time (i.e.  $\mu_{i,t} = (1+n)\mu_{i,t-1}$ ), and so the cohort aged  $I_r$  at time  $t$ , had the size  $\mu_{i,t}$  times  $(1+n)^{i-I_r}$ , at time  $t-(I_r-i)$ , or rather,

$\mu_{i,t-(I_r-i)} \times (1+n)^{I_r-i} = \mu_{i,t}$ . Similarly since wages grow over time by the rate of technological progress,  $g$ , wage earnings received must be discounted over time, and values of accumulated funds increase at the real interest rate over time. So, the left hand side of (7) shows the present value of the cohort's pension fund upon retirement, and the right hand side shows the present value of expected payouts from the fund to the cohort upon retirement.

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<sup>5</sup> A more detailed description is included in the appendix.

In model economies with full labour force participation, and in a steady state, the budget constraint for the PAYGO program changes to:

$$\sum_{i=I_w}^{i=I_r-1} \pi_i^e \mu_i p_r e_i w = \sum_{i=I_r}^{i=I} \mu_i b_i e_i w . \quad (8)$$

This is so since  $\pi^l=1$ . The budget constraint for the UFFP program changes to the following for the cohort that begins retirement:

$$\sum_{i=I_w}^{i=I_r-1} \pi_i^e \mu_i (1+n)^{i-I_r} p_r e_i w (1+g)^{i-I_r} (1+r)^{I_r-i} = \sum_{i=I_r}^{i=I} \mu_i (1+n)^{I_r-i} b_i e_i w (1+g)^{I_r-i} (1+r)^{i-I_r} . \quad (9)$$

The key item to note is that, in a steady state with complete labour force participation, the PAYGO and UFFP pension programs are identical (identical premium rates with identical benefit payouts), when  $(1+r)=(1+g)\times(1+n)$ , or when  $r\cong g+n$ . If  $r>g+n$ , then the UFFP premium rate is lower than that of the PAYGO plan. If  $r<g+n$ , then the UFFP premium rate is higher than that of the PAYGO plan.

#### IV. Model Calibration

In the model economies investigated, each period corresponds to one year. Agents have uncertain lifespan but commonly know age-specific survival rates. The maximum lifespan is set at 95 years, and all agents die before their 95<sup>th</sup> birthday ( $I=95$ ). Agents begin supplying labour and making decisions regarding consumption and asset holdings at 25 years of age ( $I_w=25$ ). Agents fully withdraw from the labour force at 65 ( $I_r=65$ ). There is no aggregate uncertainty, but there exists employment uncertainty at the individual level. Agents save to supplement income received from the government employment insurance and pension programs, to smooth consumption over the life cycle. Information pertaining to the model economies is taken from Canadian statistics for 1987. This year was chosen as a base year since it was a year when Canada was in the middle of an expansionary phase, as opposed to a recessionary or boom phase.

##### **Population**

In the model economies examined, a no-migration scenario was assumed for simplicity.<sup>6</sup> Survival rates were derived from the ratio of the population to the population plus deaths at each age. This ratio was calculated using the 1987 statistics for Canada, from the CANSIM database (Statistics Canada). Age-specific rates can be calculated for single ages from birth to 69 years of

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<sup>6</sup> This is done to eliminate complications that arise in trying to model differences in immigrant wealth holdings upon arrival and labour market characteristics in economies with employment uncertainty. See Storesletten (2000) for an illustration of modeling these differences in economies without employment uncertainty.

age. Age-specific survival rates for agents in the cohort groups, 70-74, 75-79, 80-84, 85-89 and 90 and older, were assumed to be constant for each age over the age category. The survival rate of 94 year-olds is set to 0. Fertility rates were constructed using the 1987 age-specific fertility rates of females from Statistics Canada (1993) in five-year cohorts from 15-19 years of age, to 45-49 years of age. Fertility rates were assumed constant over all ages in each cohort group. Over 1960-2000, the sex ratio of births in Canada averaged 1.055 males for every female. For the model economies, the male-female split among newborns is set to 1.055:1.

Given the Canadian population parameters from 1987, and a no-migration process, the population growth rate in the steady-state was calculated to be  $-0.79\%$ . Another model economy was calculated using the higher fertility rates of Canadian females in 1970, which results in a steady-state population growth rate of  $0.43\%$ .<sup>7</sup> For simplicity, the steady-state population distribution for males was then used for the model economies. This was done to avoid modeling female labour force participation decisions during child-rearing years. The population distribution of males under the low fertility assumption defines the population distribution for Model Economy 1. The high fertility rate defines the population distribution for Model Economy 2. These population distributions of males are compared with the distributions of the Canadian population in 1987 and 2000 in Figure 1.

### ***Preferences and Technology***

The discount rate,  $\beta$ , and the constant relative risk aversion parameter,  $\sigma$ , were set to 1.011 and 1.12, following the estimates established by Hurd (1989). The production function is a standard Cobb-Douglas with the share to capital,  $\alpha$ , set to 0.30. Depreciation is set at 0.06. The scale parameter,  $B$ , is set to 0.4, to keep the optimal asset holdings within the grid specified in the appendix. The value for  $g$ , technological progress is also set at 0.02. Table 1 presents model parameters that are fixed in the analysis.

### ***Earnings and Employment Uncertainty***

The age-specific labour endowment profile is constructed using the average earnings levels of CPP Male contributors in 1987, from Health and Welfare Canada (1989:Table 13), for agents between the ages of 25 and 64 (inclusive). The employment insurance benefit rate is set at 45%

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<sup>7</sup> The steady-state population calculation procedure is described in the appendix.

of earnings for  $i = 25$  to  $64$ .<sup>8</sup> Employment uncertainty was modeled by using the age-specific unemployment rates for males in 1987.<sup>9</sup> Agents in the model economy are either employed, unemployed, or not in the labour force. All labour force participants are willing to work, but those who are unemployed cannot work due to a shortage of demand, and wages do not adjust to clear the market. The probability of being unemployed in a given period for an agent is unrelated to the probability of being unemployed in any other period. The moral hazard problem resulting from the existence of an insurance program is not modeled in the exercise (see Hansen and Imrohoroglu (1992)).

To keep the exercise manageable, a fixed proportion of individuals in each age cohort will not participate in the labour force, and will continue to do so for their lifetimes. All labour force participants will remain in the labour force for their working lifetimes (and have status as employed or unemployed). There is no movement into or out of the labour force by individuals between the age of 25 and 64.

Over the last two decades in Canada, the labour force participation rate of males between the ages of 25 and 54 was approximately 93%.<sup>10</sup> One set of model economy results corresponds to an economy with a labour force participation rate fixed at 93% for agents of working age. Since 1980, the labour force participation rate of all males and females between the ages of 25 and 54 rose from 77% to 86%, while that of both males and females between the ages of 15 and 64 rose from 68% to 78%. Another set of model economy results corresponds to an economy with a labour force participation rate fixed at 70% for agents of working age.

### **Government Programs**

The model takes as given that agents retire at the age of 65, whether or not a pension program exists. The PAYGO pension program is of the pay-as-you-go type where current contributions are used to fund current benefits. In the steady state, total benefits will equal total contributions in each period. The UFFP pension program is one that is fully funded for each age cohort, and also pays out benefits to those aged 65 and older, including those who had never participated in the labour force (i.e. they received welfare benefits before age 65).

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<sup>8</sup> The average weekly benefit was approximately 45% of the average weekly earnings over 1972-1994 (Statistics Canada (1995: 50)).

<sup>9</sup> Figures from CANSIM database.

<sup>10</sup> The labour force participation rate of males between the ages of 55 and 64 is much lower; as an example, it was 65.8% in 1987. It is apparent that many individuals take, or are forced into, early retirement before the age of 65. The model economies investigated in this study do not allow early or late retirement.

The pension benefit profile is derived from the average amount of monthly benefits put into play for males, in 1987, of \$305, an annual amount of \$3,660 (Health and Welfare Canada (1987)). This amount will also serve as the annual welfare benefit to those not in the labour force, and will be reduced by the income tax rate to maintain a balance between pension benefits and welfare benefits, since pension benefits are taxable, but welfare will not be subject to income tax.

A complication arises in setting both welfare and pension rates over time in real terms. In a steady state economy with rising real wages, a pension program that pays out benefits that are fixed in real terms would cause the pension payroll tax rate to fall over time, and render the pension program irrelevant in an economy with technological progress. Similarly for the welfare program, if welfare payments are fixed in real terms, over time, the payments will fall in relative terms compared to wages. The employment insurance program pays out benefits that grow over time in real terms since benefits are set to 45% of the value of real wages. In order to examine steady state equilibria, pension and welfare payments must increase in real terms over time. In a welfare program that is financed by general taxes, welfare benefits will increase in real terms at the rate of technological progress each year. In a similar fashion, pension benefits will also increase in real terms at the rate of technological progress each year. The endowment level upon retirement,  $e_{65}$ , is set to the annual amount \$3,660, and the benefit rate,  $b_i$ , is set to 1 for  $i = 65$  to 94. The endowment and pension benefit profile is depicted in Figure 2. The welfare benefit level is set to the after-tax pension benefit level.

The government also levies a flat tax rate equal to 40% or 20% of total income, from labour earnings net of payroll taxes, returns on assets, pension or employment insurance benefits, and from the transfer of accidental bequests. The flat rate is used for simplicity to avoid the complications that arise in modeling progressive tax systems, including tax deferrals on some forms of retirement savings. Government expenditures as a percentage of gross domestic product in major industrialized nations varied from a low of 33.0% in the US in 1994, to 68.7% in Sweden.

## V. Model Economy Results

The results are presented for economies with two different population age distributions and population growth rates. The first model economy has a negative growth rate, -0.79% per year, due to fertility rates below the replacement level, and 28% of the population aged 25 years

and over are considered retirees (aged 65 years or older). The second model economy has a positive population growth rate, 0.43% per year, and 16% of the population aged 25 years and over are considered retirees. The parameters that are calibrated (set in advance), but will vary are: a) the real interest rate (either 2% or 4%); b) the labour force participation rate (either 93% or 70%); and c) the income tax rate (either 40% or 20%). All values in the tables that are calibrated parameters (set in advance) are in boldface.

The tables present the calculated pension payroll premium rates, net savings rates, and foreign capital inflow rates, along with the value for expected lifetime utility when agents enter the labour force. For model economies with the UFFP program, the ratio of pension fund assets to total capital stock is also calculated, and indicates to what extent the UFFP fund can provide funds for production. In addition, for model economies with the PAYGO pension program, the tax burden of welfare payments of the elderly who were never employed is calculated as a percentage of total income taxes received. This will give an indication as to how much in tax revenues the elderly receive. This is important to note because, in the UFFP case, these payments are made from the pension program itself, rather than through general income tax revenues, and so the UFFP pension payroll tax rate will be higher when there is a subset of the population who never participated in the labour force. So, it will be important to examine how a decline in labour force participation affects pension premium rates and expected lifetime utility or welfare, without any offsetting change in overall income tax rates.

The results for Model Economy 1, with a negative population growth rate, are presented in Tables 2 ( $r = 2\%$ ) and 3 ( $r = 4\%$ ). The results for Model Economy 2, with a positive population growth rate, are presented in Tables 4 ( $r = 2\%$ ) and 5 ( $r = 4\%$ ). Three key items are important to highlight at this point. First, the PAYGO pension premium rate does not change when the labour force participation rate changes, because the PAYGO plan only covers labour force participants upon retirement. On the other hand, since the UFFP plan covers all members of the elderly population, the UFFP premium rate does increase when the labour force participation rate falls. Second, the PAYGO pension premium rate does not change when the real interest rate changes, but the UFFP premium rate does. Under the UFFP program, pension benefits are fully funded, and the pension fund accumulates a balance over the lifetime of the cohort. The higher the real interest rate, the faster the fund accumulates value. Since the UFFP program is set up to achieve a target fund value over time, the higher the interest rate, the lower the contribution level required to achieve the target fund value. Therefore, when the interest rate rises, the contribution rate declines. Third, The UFFP premium tax rate does not vary between

model economies, all else equal. This indicates that the UFFP program is not sensitive to fertility rate changes, while the PAYGO pension program is.

### ***Model Economy 1***

In column 2 of the five-column Table 2, the real interest rate is 2%, the labour force participation rate is 93%, and the income tax rate is 40%. The PAYGO pension premium rate for Model Economy 1 is calculated to be 8.89% of wage earnings, and the figure for the value function of the individual upon entering the labour force (the value for expected lifetime utility) is -435.54. For the same set of fixed parameter values, the UFFP pension premium rate is calculated to be 7.54% of wage earnings, and the value of expected lifetime utility is -434.88. This indicates that labour market participants are better off under the UFFP program than under the PAYGO program (all else equal). The same can be said when the income tax rate is changed to 20%, since the values for expected lifetime utility are -419.05 under the PAYGO plan, and -418.39 under the UFFP plan, as indicated in column 3. The results in columns 4 and 5, when the interest rate is 2% and the labour force participation rate is 70%, suggest that agents are worse off under the UFFP program than under the PAYGO program: the expected lifetime utility of labour force participants is lower under the UFFP plan than under the PAYGO plan. However, this is to be expected because the premium rate for the UFFP plan is 10.02%, higher than the 8.89% rate for the PAYGO plan. In the UFFP environment, the government has taxation revenues that it may devote to programs other than supporting the welfare of the elderly. In the PAYGO environment, over five percent of taxation revenues are used to support the consumption of elderly individuals who never participated in the labour force. The UFFP program covers these individuals. It becomes apparent in this case that the outcomes under the two different programs are not directly comparable when the income tax rate does not adjust to increases in the pension premium rate under the UFFP to compensate for an increased amount of the elderly who never participated in the labour force.

When the real interest rate is set at 4% for Model Economy 1 (see Table 3), the premium rates under the UFFP program are lower than those of the PAYGO plan, under all scenarios listed (labour force participation rates equal to 93% or 70%, and income tax rates equal to 40% or 20%). As a result, the welfare of labour force participants is higher in the UFFP environment than in the PAYGO environment, in all cases listed. This indicates that when the labour force participation rate drops to 70%, the additional pension benefits paid out to the elderly who never

worked is not sufficient to raise the UFFP pension premium rates to that of the PAYGO plan. The value of the pension fund grows quickly because the real interest rate is 4%.

The tables also provide savings rates and net foreign capital inflow rates for the model economies. The results in Tables 2 and 3 show that, in an environment with a PAYGO plan, savings rates are quite low when the real interest rate is 2%, and the economies require foreign capital inflows to meet capital stock requirements. Certainly, a lower income tax rate helps increase the savings rate since the after-tax return on savings will be larger. However, under the same real interest rate in the UFFP environment, the fully-funded nature of the pension program increases the domestic savings rate, causing less reliance on foreign investors. The model economy in the PAYGO pension environment requires a higher real interest rate, than when under the UFFP environment, to avoid reliance on foreign investors. It is only when the real interest rate rises to 4% that the model economy in the PAYGO environment requires little foreign capital. With this higher interest rate, the model economy is a net capital importer when the tax rate is 40%, and the model economy is a net exporter of capital funds when the income tax rate is 20%. Under the UFFP plan, the model economies are large net exporters of capital funds, under all four scenarios with a real interest rate of 4%.

### ***Model Economy 2***

The population of Model Economy 2 has a positive growth rate, and a lower proportion of retirees, when compared to the population of Model Economy 1. As a consequence, the PAYGO pension premium rate is only 6.17% for Model Economy 2, while it is 8.89% for Model Economy 1. The main attraction of the PAYGO program when public pensions were first developed was that PAYGO pension premium rates are low when the population grows. On the other hand, the UFFP pension rates are the same as those for Model Economy 1 because the UFFP program is fully funded. The main attraction now for fully-funded pension programs is that the premium rates do not depend on fertility rate differences in model economies. The modeling results show this clearly.

The results presented in Table 4 indicate that, with a real interest rate of 2%, a productivity growth rate of 2%, and a positive population growth rate, the PAYGO pension premium rate (6.17%) is lower than the UFFP pension premium rates (7.54% or 10.02%). As such, individuals in the PAYGO environment are better off than those in the UFFP environment: the expected lifetime utility values for agents in the PAYGO pension environment are larger than the corresponding values for agents in the UFFP environment when the real interest rate is 2%.

However, when the real interest rate is 4%, the results in Table 5 indicate that agents are better off under the UFFP environment rather than the PAYGO pension environment, because the pension fund accumulates over time at the higher interest rate of 4%: Model Economy 2 agents in the UFFP environment pay lower premium rates than agents in the PAYGO pension environment.

When the real interest rate is low, model economies in the PAYGO environment require substantial flows of foreign capital to meet investment requirements, significantly more so than model economies in the UFFP environment. The calculated savings rates for Model Economy 2 indicate that foreign capital dependence isn't eliminated until the real interest rate is about 4% when the economy has a PAYGO pension program. Foreign capital dependence for Model Economy 2 under the UFFP program is eliminated when the real interest rate is about 2%.

### ***Discussion***

Model economies have been compared under PAYGO and UFFP pension environments, by holding many parameters constant. Both pension systems have been setup to provide the same stream of pension income to beneficiaries of a given age cohort. There are three major differences between the two programs. One deals with the welfare funding of the elderly, and the other deals with the effect of pension systems on the foreign capital requirements, or the real interest rate, and the last deals with how the two programs respond to changes in fertility rates.

The first major difference between the two systems is that the UFFP plan pays out benefits to members of a given cohort who never participated in the labour force, whereas these members received welfare benefits paid by general income tax revenues in the PAYGO pension system. In cases when the labour force participation rate decreases, the UFFP pension premium rate must rise to cover the pension benefits of those who do not enter the labour force. However, a larger difference between  $r$  and  $g+n$  can lighten the additional funding requirements of individuals who do not enter the labour force and pay into the pension fund. In the case when  $r = 4\%$  and  $g+n \approx 2\%$ , the decline in the labour force participation rate to 70% was not enough to raise the UFFP pension premium rate above that of the PAYGO plan. When the real interest rate was 2% and the labour force participation rate declined from 93% to 70%, the UFFP premium tax rate rose above the PAYGO premium rate, but there was no corresponding decline in the income tax rate since the UFFP program shifts the welfare burden of the elderly, who never participated in the labour force, from general tax revenues (PAYGO plan) to the pension system (UFFP plan). To get a better comparison between the two plans, the model economies should be

examined under a scenario whereby the increased pension premium rate in the UFFP plan should be perfectly offset by a reduction in income tax rates, as the government no longer provides welfare benefits to the elderly through general income tax revenues.

The second important difference between the two plans is the potential effect of these pension programs on interest rates. Since the UFFP program is fully-funded, a pension fund exists. The value of the fund can be substantial. In the scenarios investigated in this study, the value of the pension fund varied between forty and sixty-five percent of total domestic capital. Under a closed economy assumption, the real interest rate for the economy under a UFFP environment would be about 2%, whereas the real interest rate for the economy in a PAYGO pension environment would be about 4%. So, it might not be appropriate in all cases to compare the two programs without accounting for the effects of pension funds on interest rates.

The model economies were investigated in the steady state, and did not experience a demographic transition. In light of this, can one make any prediction as to how these public programs will have to adjust to demographic changes? One may conclude, given the results presented in this paper, that changes in fertility will not have any effect on the UFFP program, but will have an impact on the PAYGO plan. If the fertility rate falls, the PAYGO pension rates must rise to maintain a balance between premia and benefits. No such adjustment needs to be made for the UFFP plan since it is fully-funded for each cohort by that same cohort – there are no intergenerational concerns. However, if a demographic change occurs as a result of changing survival rates and life expectancy, it is expected that both pension plans will need some adjustment, either by increasing contribution rates to fund a longer period of benefits for survivors, or decreasing the benefit payouts.

### ***Directions for Future Work***

Future work should examine the three items mentioned above, namely the adjustment of income tax rates to examine pension systems that have no differential effects on government spending, the effect of pension systems on the interest rate and closed economy comparisons, and changes in survival rates on pension systems. In addition, transitional environments may be investigated in overlapping generations models. The PAYGO public pension programs in the developed nations are in a state of crisis because of population aging. People are living longer and retiring earlier, while fertility rates have fallen to below-replacement levels. It would be interesting to know how the PAYGO program would fare relative to the UFFP program during a demographic transition, and to model demographic change while accounting for changes in capital-labour

ratios, wages and rates of return. As the population ages, especially in light of the baby-boom experience of some nations, it is expected that the return on capital will fall, and wages will rise. If the pace of technological change also accelerates, we may be able to weather the demographic storm without major adverse effects on pension benefits. As demonstrated in Emery and Rongve (1999), the expected demographic changes may have some advantages on wages of the young smaller cohorts for which we are not fully accounting. This study has also not taken into account business cycles and how they affect rates of return on assets. Would changes in rates of return on assets have a large impact on certain cohorts in the economy with the UFFP program?

## **VI. Conclusion**

Many of the industrialized nations have experienced declines in fertility rates, declines in labour force growth, and increases in life expectancy and survival rates across age groups. Each of these has contributed to the concern about the sustainability of the “pay-as-you-go” (PAYGO) public pension program. Economies are facing a fall in the worker to retiree ratio, and are looking at ways to reform the pension system. In order to sustain the PAYGO system in the coming years, either (i) contribution rates must increase, (ii) benefit levels must decrease, (iii) the age of retirement must increase, (iv) general tax rates must rise, or (v) some combination of these four options must be enacted. Other pension reform ideas include abandoning the PAYGO system in favor of public fully-funded plans, like the universal fully-funded pension plan (UFFP). This paper has examined some of the aspects of the public PAYGO and UFFP plans and has attempted to compare model economies in the two different pension environments.

An overlapping generations general equilibrium model has been used to examine steady-state equilibria of model economies with different population structures along with the two different public pension systems. The modeling environment was somewhat limited in its richness due to the complications in computing equilibria, but nonetheless can offer answers to some questions of interest to policymakers considering pension reform. First, the UFFP program, since it is fully funded, generates substantial amounts of capital available for production, whereas the PAYGO plan does not provide funds for investment. As a result, economies with a PAYGO plan are either much more susceptible to foreign capital dependence, or have a higher equilibrium real interest rate, when compared to economies with the UFFP program. Second, the PAYGO program is a more favorable program than the UFFP program, in general, as long as the real interest rate is less than the sum of the growth rates of labour earnings in real terms (or the

sum of the growth rates of technological progress,  $g$ , and the labour force,  $n$ ), but the real interest rate in a closed economy is not independent of the type of public pension program. Third, UFFP pension payroll tax rates are not dependent on fertility or labour force growth rates, whereas PAYGO pension payroll tax rates are dependent on such growth rates. Low labour force growth rates cause high PAYGO payroll tax rates. Fourth, the UFFP pension payroll tax rate is dependent on the real interest rate, whereas the PAYGO pension payroll tax rates are not. Low real interest rates cause high UFFP payroll tax rates.

The results presented in this paper do give us an indication of relative welfare levels of agents in the different pension environments, but the simulations are not advanced or complete enough to provide clear answers to some welfare-related questions. Much work remains to be done in refining the overlapping generations model economies used in the simulations for the purpose of this paper. This paper has not attempted to model transitional economies, the effect of uncertainty in asset returns, the effect of different pension systems on closed economy outcomes and general income tax systems.

## References

- Abel A.B. (1987). Capital accumulation and uncertain lifetimes with adverse selection. *Econometrica* 54: 1079-1097.
- Auerbach, A.J., and Kotlikoff, L.J. (1987), *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.
- Bohn, H. (1999), "Should the Social Security Trust Fund Hold Equities? An Intergenerational Welfare Analysis," *Review of Economic Dynamics* 2: 666-697.
- Emery, J.H. and Rongve, I (1999), "Much Ado About Nothing? Demographic Bulges, the Productivity Puzzle and CPP Reform," *Contemporary Economic Policy* 17(1): 69-79.
- Feldstein, M.S. (1974), "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy* 82: 905-26.
- Finance Canada (1999), "Canada Pension Plan, Steady-State Financing." Available at [http://www.cpp-rpc.ca/backgr/19991209bk1\\_e.html](http://www.cpp-rpc.ca/backgr/19991209bk1_e.html).

- Hansen, G.D., and Imrohoroglu, A. (1992), "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard." *Journal of Political Economy* 100 (1): 118-142.
- Health and Welfare Canada (1987), *Monthly Statistics, Income Security Programs*. Ottawa .
- Health and Welfare Canada (1989), *Canada Pension Plan Contributors, 1987*. Ottawa.
- Ho L.S. (1997). A universal fully funded pension scheme. *Contemporary Economic Policy* 15: 13-20.
- Hubbard, R.G., and Judd, K.L. (1987), "Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax." *The American Economic Review* 77 (4): 630-646.
- Hubbard, R.G., Skinner, J. and Zeldes, S.P. (1995), "Precautionary Saving and Social Insurance." *Journal of Political Economy* 103 (2): 360-399.
- Huggett, M. (1996), "Wealth distribution in life-cycle economies," *Journal of Monetary Economics* 38: 469-494.
- Huggett, M. and Ventura, G. (1999), "On the Distributional Effects of Social Security Reform," *Review of Economic Dynamics* 2: 498-531.
- Hurd, M.D. (1989), "Mortality Risk and Bequests," *Econometrica* 57, 4: 779-814.
- Rios-Rull J.V. (2001). Population changes and capital accumulation: The aging of the baby boom. *The B.E. Journals in Macroeconomics* 1, Article 7.
- Rosen, H.S., Boothe, P., Dahlby, B., and Smith, R.S. (1999), *Public Finance in Canada*. McGraw-Hill Ryerson Limited, Toronto.
- Statistics Canada (1993), *Selected Birth and Fertility Statistics, Canada, 1921-1990*. Ottawa, Catalogue 82-553.
- Statistics Canada (1995), *Unemployment Insurance Statistics*. Ottawa, Catalogue 73-202S.
- Storesletten, K. (2000), "Sustaining Fiscal Policy Through Immigration," *Journal of Political Economy* 108: 300-323.
- Wilson S.J. (2001). Retirement savings and public pensions. Department of Economics Working Paper No.98, University of Regina.
- Wilson S.J. (2003). A dynamic general equilibrium analysis of migration and capital formation: The case of Canada. *Review of Economic Dynamics* 6: 455-481.

## Appendix: Description of the OLG Modeling Framework

The variables are transformed as follows to remove the effects of growth in the population and of technology:

$$\begin{aligned}\tilde{L}_t &= L_t / L_t, \tilde{K}_t = K_t / L_t A_t, \tilde{T}_t = T_t / A_t, \tilde{a}_t = a_t / A_t, \tilde{c}_t = c_t / A_t, \tilde{\omega}_t = \omega_t / A_t, \\ \tilde{\mu}_t &= \mu_t / L_t, \tilde{N}_t = N_t / L_t.\end{aligned}$$

In the steady state, the transformed variables for capital and labour inputs, for transfers and for the age-distribution of agents in the economy are constant over time. The untransformed variables therefore all grow at constant rates.

In the small open economy framework the return on capital is determined on international markets. Steady-state equilibria are computed using the following algorithm:

- (1) There is no aggregate uncertainty. Calculate the expected value of the aggregate labour supply in efficiency units using the age-specific labour force participation and unemployment rates ( $\tilde{L} = \sum_i \pi_i^e (\pi^l \tilde{\mu}_i) e_i$ ).
- (2) The value of labour supply along with the real interest rate defines the value of aggregate capital stock and total output. Calculate the wage rate per efficiency unit.  $\tilde{\omega} = F_2(\tilde{K}, \tilde{L})$  and  $R = 1 + F_1(\tilde{K}, \tilde{L}) - \delta$ , with  $R$  set on international markets, and here assumed constant.
- (3) Given the employment insurance and pension benefit rates, and the expected number of unemployed agents and the number of pensioners of each age cohort, calculate the premium rates for each program (Eqs (4-5) and (7)).
- (4) Guess the value for the average transfer of accidental bequests,  $\tilde{T}$ .
- (5) Calculate the optimal decision rules,  $\tilde{c}$ ,  $\tilde{a}'$ , of all agents aged  $I_w$  (25) to  $I$  (94), with the values of  $\tilde{a}_{I_w-1}$  and  $\tilde{a}_{I+1}$  set to zero. To do this, the individual's dynamic programming problem must be solved:

$$V(\tilde{a}, l, i) = \max_{(\tilde{c}, \tilde{a}')} u(\tilde{c}) + \beta(1+g)^{1-\sigma} s(i) E[V(\tilde{a}', l', i+1) | (\tilde{a}, l, i)],$$

subject to,

- (a)  $\tilde{c} + \tilde{a}'(1+g) \leq \tilde{a} + (r\tilde{a} + (1-p_e - p_r)\tilde{\omega}l(i)e(i) + b(i)\tilde{\omega}(1-l(i))e(i) + \tilde{T}) \times (1-\tau)$
- (b)  $\tilde{c} \leq 0, \tilde{a}' \leq 0, \tilde{a}' = 0$  if  $i = I_w-1$  or  $I$ .
- (c)  $l(i) = 1$  with prob =  $\pi_i^e$  and 0 with prob  $1 - \pi_i^e$ .

The decision rules are calculated by maximizing the value function on gridpoints defined over the state space. 1200 evenly spaced gridpoints were used for the asset variables  $a$  and  $a'$ , and two gridpoints were used for  $l$ . For each period of an individual's life, the value function was calculated on 2.9 million gridpoints. The optimal value of  $a'$  for a given value of  $a$  and  $l$  was that value where the value function was maximized.

- (6) Construct population samples with random labour characteristics using the age-specific labour endowments and employment probabilities. Calculate the asset holdings of each agent of the sample, given his employment history and the optimal level of asset holdings given step (3). For each population sample, calculate the value of accidental bequests per adult survivor,  $\tilde{T}$ , from a random sample of agents from each cohort who die in a given period, for each population sample, and find the average over samples.
- (7) If the average value of  $\tilde{T}$  calculated in step (4) differs from the initial guess in step (4), repeat the procedure in steps (4-6) using the result from step (6) as the initial guess until convergence is achieved.
- (8) Once convergence is achieved, calculate the net savings rate, net foreign capital inflow rate, and additional variables for the economy.

The population samples were constructed using the steady-state population distribution for the model economy. Three hundred samples of twenty-six thousand agents were used to find the average values in step (4). These calculations were completed using OX version 2.2 by J.A. Doornik.

Table 1: Fixed Model Economy Parameters

$\alpha$	$\beta$	$\delta$	$\sigma$	$I_w$	$I_r$	$I$	$b_{20-64}$	$b_{65-94}$	$s$
0.3	1.011	0.06	1.12	25	65	94	0.45	1	1987 rates, Canada

Table 2: Model Economy 1 Results, Real Interest Rate = 2% (all table figures in percentages)

<b>Population Growth Rate</b>	<b>-0.79</b>			
<b>Elderly (&gt;64)/Adult Population (&gt;24)</b>	<b>27.63</b>			
<b>Aggregate Unemployment Rate</b>	<b>6.83</b>			
<b>Employment Insurance Payroll Tax Rate</b>	<b>3.24</b>			
<b>Productivity Growth Rate</b>	<b>2.00</b>			
<b>Real Interest Rate</b>	<b>2.00</b>			
<b>Labour Force Participation Rate</b>	<b>93</b>	<b>93</b>	<b>70</b>	<b>70</b>
<b>Income Tax Rate</b>	<b>40</b>	<b>20</b>	<b>40</b>	<b>20</b>
<b>PAYGO Pension Program Results</b>				
Pension Payroll Tax Rate	8.89			
Net Savings Rate	1.85	2.83	1.86	2.86
Foreign Capital Inflow Rate	2.89	1.83	2.88	1.80
Welfare Payments of the Elderly/Taxation Income	0.94	3.42	5.36	13.82
V(0) – Expected Utility when entering Labour Force	-435.54	-419.05	-435.99	-419.69
<b>UFFP Pension Program Results</b>				
Pension Payroll Tax Rate	7.54		10.02	
Net Savings Rate	3.95	4.88	4.54	5.42
Foreign Capital Inflow Rate	0.62	-0.39	-0.02	-0.97
UFFP Pension Fund Assets/Total Capital Stock	47.00	47.00	62.44	62.44
V(0) – Expected Utility when entering Labour Force	-434.88	-418.39	-436.55	-420.25

Note: The difference in the values of expected utility in columns 2 and 4, and in columns 3 and 5, are due to differences in the value of the transfer of accidental bequests for both PAYGO and UFFP programs, and due to the increased pension premium rate in the case of the UFFP program, since all other parameters for the individual optimization problem are held constant between these two sets of corresponding columns. With fewer labour force participants, and thus fewer savers, when the participation rate drops from 93% to 70%, so too does the annual transfer, and consequently the value of expected lifetime utility falls. Similarly, when the labour force participation rate drops, the UFFP premium rate rises, and consequently, the value of expected lifetime utility also falls.

Table 3: Model Economy 1 Results, Real Interest Rate = 4% (all table figures in percentages)

<b>Population Growth Rate</b>	<b>-0.79</b>			
<b>Elderly (&gt;64)/Adult Population (&gt;24)</b>	<b>27.63</b>			
<b>Aggregate Unemployment Rate</b>	<b>6.83</b>			
<b>Employment Insurance Payroll Tax Rate</b>	<b>3.24</b>			
<b>Productivity Growth Rate</b>	<b>2.00</b>			
<b>Real Interest Rate</b>	<b>4.00</b>			
<b>Labour Force Participation Rate</b>	<b>93</b>	<b>93</b>	<b>70</b>	<b>70</b>
<b>Income Tax Rate</b>	<b>40</b>	<b>20</b>	<b>40</b>	<b>20</b>
<b>PAYGO Pension Program Results</b>				
Pension Payroll Tax Rate	8.89			
Net Savings Rate	2.94	4.89	2.95	4.88
Foreign Capital Inflow Rate	0.77	-1.45	0.76	-1.43
Welfare Payments of Elderly/Taxation Income	0.85	2.00	4.62	11.61
V(0) – Expected Utility when entering Labour Force	-436.72	-417.50	-437.43	-418.63
<b>UFFP Pension Program</b>				
Pension Payroll Tax Rate	4.17		5.53	
Net Savings Rate	4.52	6.35	4.92	6.65
Foreign Capital Inflow Rate	-1.03	-3.11	-1.48	-3.44
UFFP Pension Fund Assets/Total Capital Stock	45.31	45.31	60.19	60.19
V(0) – Expected Utility when entering Labour Force	-434.34	-415.12	-435.73	-416.95

Table 4: Model Economy 2 Results, Real Interest Rate = 2% (all table figures in percentages)

<b>Population Growth Rate</b>	<b>0.43</b>			
<b>Elderly (&gt;64)/Adult Population (&gt;24)</b>	<b>15.78</b>			
<b>Aggregate Unemployment Rate</b>	<b>6.93</b>			
<b>Employment Insurance Payroll Tax Rate</b>	<b>3.28</b>			
<b>Productivity Growth Rate</b>	<b>2.00</b>			
<b>Real Interest Rate</b>	<b>2.00</b>			
<b>Labour Force Participation Rate</b>	<b>93</b>	<b>93</b>	<b>70</b>	<b>70</b>
<b>Income Tax Rate</b>	<b>40</b>	<b>20</b>	<b>40</b>	<b>20</b>
<b>PAYGO Pension Program Results</b>				
Pension Payroll Tax Rate	6.17			
Net Savings Rate	3.37	5.19	3.39	5.22
Foreign Capital Inflow Rate	6.19	4.22	6.17	4.19
Welfare Payments of the Elderly/Taxation Income	0.66	1.69	3.75	9.72
V(0) – Expected Utility when entering Labour Force	-434.52	-418.18	-434.93	-418.75
<b>UFFP Pension Program Results</b>				
Pension Payroll Tax Rate	7.54		10.02	
Net Savings Rate	6.80	8.46	7.80	9.38
Foreign Capital Inflow Rate	2.49	0.68	1.40	-0.31
UFFP Pension Fund Assets/Total Capital Stock	39.85	39.85	52.94	52.94
V(0) – Expected Utility when entering Labour Force	-435.19	-418.84	-436.80	-420.60

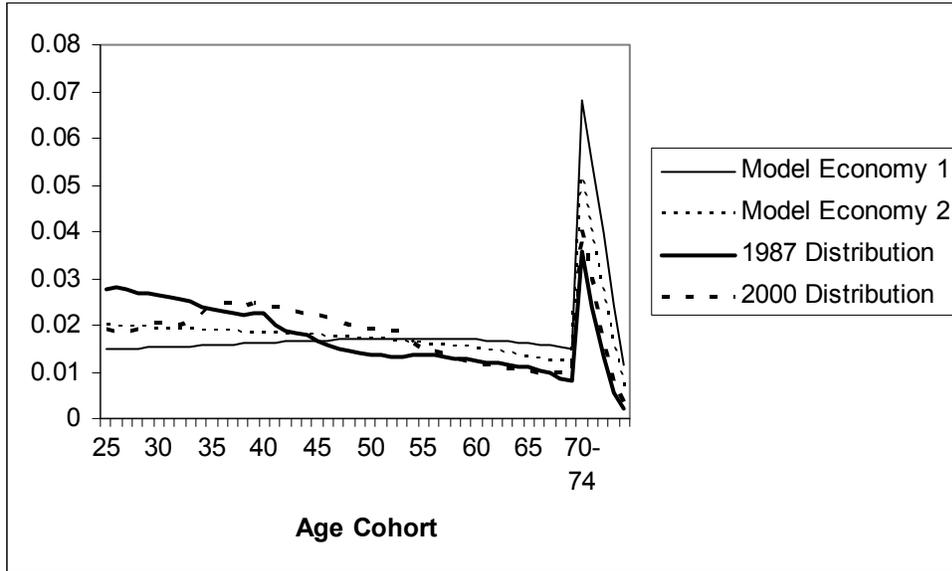
Table 5: Model Economy 2 Results, Real Interest Rate = 4% (all table figures in percentages)

<b>Population Growth Rate</b>	<b>0.43</b>			
<b>Elderly (&gt;64)/Adult Population (&gt;24)</b>	<b>15.78</b>			
<b>Aggregate Unemployment Rate</b>	<b>6.93</b>			
<b>Employment Insurance Payroll Tax Rate</b>	<b>3.28</b>			
<b>Productivity Growth Rate</b>	<b>2.00</b>			
<b>Real Interest Rate</b>	<b>4.00</b>			
<b>Labour Force Participation Rate</b>	<b>93</b>	<b>93</b>	<b>70</b>	<b>70</b>
<b>Income Tax Rate</b>	<b>40</b>	<b>20</b>	<b>40</b>	<b>20</b>
<b>PAYGO Pension Program Results</b>				
Pension Payroll Tax Rate	6.17			
Net Savings Rate	5.38	8.95	5.39	8.94
Foreign Capital Inflow Rate	2.15	-1.90	2.14	-1.89
Welfare Payments of Elderly/Taxation Income	0.60	1.44	3.44	8.32
V(0) – Expected Utility when entering Labour Force	-435.84	-416.95	-436.45	-417.93
<b>UFFP Pension Program</b>				
Pension Payroll Tax Rate	4.17		5.53	
Net Savings Rate	7.91	11.26	8.59	11.78
Foreign Capital Inflow Rate	-0.72	-4.52	-1.50	-5.11
UFFP Pension Fund Assets/Total Capital Stock	37.73	37.73	50.13	50.13
V(0) – Expected Utility when entering Labour Force	-434.85	-415.97	-436.13	-417.62

**Table 6: Comparison of Model Economy Results**

<b>Model Economy</b>	<b>1</b>		<b>2</b>	
<b>Population Growth Rate</b>	<b>-0.79</b>		<b>0.43</b>	
<b>Elderly (&gt;64)/Adult Population (&gt;24)</b>	<b>27.63</b>		<b>15.78</b>	
<b>Aggregate Unemployment Rate</b>	<b>6.83</b>		<b>6.93</b>	
<b>Employment Insurance Payroll Tax Rate</b>	<b>3.24</b>		<b>3.28</b>	
<b>Productivity Growth Rate</b>	<b>2.00</b>		<b>2.00</b>	
<b>Real Interest Rate</b>	<b>2.00</b>	<b>4.00</b>	<b>2.00</b>	<b>4.00</b>
<b>Labour Force Participation Rate</b>	<b>93</b>			
<b>Income Tax Rate</b>	<b>40</b>			
<b>PAYGO Pension Program Results</b>				
<b>Pension Payroll Tax Rate</b>	<b>8.89</b>		<b>6.17</b>	
<b>Net Savings Rate</b>	<b>1.85</b>	<b>2.94</b>	<b>3.37</b>	<b>5.38</b>
<b>Foreign Capital Inflow Rate</b>	<b>2.89</b>	<b>0.77</b>	<b>6.19</b>	<b>2.15</b>
<b>Welfare Payments of Elderly/Taxation Income</b>	<b>0.94</b>	<b>0.85</b>	<b>0.66</b>	<b>0.60</b>
<b>V(0) – Expected Utility when entering Labour Force</b>	<b>-435.54</b>	<b>-436.72</b>	<b>-434.52</b>	<b>-435.84</b>
<b>UFFP Pension Program</b>				
<b>Pension Payroll Tax Rate</b>	<b>7.54</b>	<b>4.17</b>	<b>7.54</b>	<b>4.17</b>
<b>Net Savings Rate</b>	<b>3.95</b>	<b>4.52</b>	<b>6.80</b>	<b>7.91</b>
<b>Foreign Capital Inflow Rate</b>	<b>0.62</b>	<b>-1.03</b>	<b>2.49</b>	<b>-0.72</b>
<b>UFFP Pension Fund Assets/Total Capital Stock</b>	<b>47.00</b>	<b>45.31</b>	<b>39.85</b>	<b>37.73</b>
<b>V(0) – Expected Utility when entering Labour Force</b>	<b>-434.88</b>	<b>-434.34</b>	<b>-435.19</b>	<b>-434.85</b>

Figure 1: Population Distribution Across Age Cohorts



Note: Age cohorts are in single years from 25 to 69, then in 5-year groups for 70-74, 75-79, 80-84, 85-89, and 90-94.

Figure 2: Earnings and Pension Profiles for the Model Economies

