

# **How to Better Run a Capital Project: Being a Principal or a Managing Sponsor?**

David Hanly<sup>1</sup>  
and  
Keith C.K. Cheung

July 2001

## **ABSTRACT**

Project financing focuses security and liability provisions on the specific investment project. It represents an alternative to financing new investment through equity, retained earnings or debt connected to the financial position of the parent firm. The parent firm no matter what the financing arrangement still manages the investment project but with project financing, outside investors directly assume some of the project risk. This paper examines the factors that favour project financing. The analysis focuses on the advantages of risk-reduction for the parent firm from project financing versus incentives for managerial effort. It is shown with other factors equal that project financing is favoured when fixed capital costs are high relative to other costs or when the return to the project is less sensitive to unobservable levels of managerial effort.

---

<sup>1</sup>Corresponding author: Saskatchewan Energy and Mines, 2101 Scarth Street, Regina, Saskatchewan, Canada S4P 3V7. E-mail address: dhanly@sem.gov.sk.ca

CLASSIFICATION: Business Economics (M1), Corporate Finance (G3)

## **I. INTRODUCTION**

Demand for project finance has never been greater. Governments and private sectors both use it extensively to undertake capital investments. Project financing has been used to carry out investments as diverse as power stations, highways, tunnels, and hotels. Project financing is defined here as a parent company operating a new investment opportunity as a separate financial entity with income flows and liabilities treated separately from the parent company. With project financing, substantial funding for the new investment project comes from outside investors whose lending is secured by the income and the assets of the investment project and are not directly connected to the parent company's financial circumstances. The parent company gains by transferring its risk exposure on the investment project since the outside investor has limited recourse to make claims on the firm's other business activities. To its advantage, the outside lender need not be as concerned about the risk of the firm's other business activities outside the investment project. In general, when lending to an investing firm, except where lending is fully secured by specific assets, the lender is exposed to the risk of a firm's entire business activities.

With no or limited recourse against the parent company, project financing allows the parent company to develop potential income opportunities available to it at a lower risk. Though interesting, concerns such as whether and when the parent company might decide to buy out the interests of outside investors in the project will not be addressed in this paper. With project financing, outside investors become

owners in the investment project since their income is directly dependent on the financial success of the project.<sup>1</sup>

Much of the discussion on project financing so far has focussed on risk mitigation. Recent contributions include Kensinger & Martin (1988) developing new financing techniques and structures, Thomadakis and Usmen (1991) proposing optimal capital mix with project financing, Chehayl & Berger (1994) arguing that financing arrangements are a race against time, Cenatempo (1996) examining the sponsorship quality, Brealey, Cooper & Habib (1996) reviewing the usage within public sectors, and Rose (1998) highlighting necessary conditions for success.

Interestingly, attention is on funding issues from the lenders' perspectives. Rather than focusing on the outside investor, we consider the problem from the parent company perspective of whether it should finance internally or resort to project finance. One of the fundamental choices a firm must make is the ownership structure that it employs in attempting to capture gains from business opportunities it has identified. To provide a clear example, our maintained assumption is that no matter what the ownership structure the parent company will manage the new business venture. On first appearances, the decision is purely financial. The company can use internal capital resources of the firm to develop the venture and face the entire project specific risk. Alternatively, it can use project financing and have an outside

---

<sup>1</sup>The so-called build-operate-transfer (BOT) model is a common practice among project financing. However, our analysis is based on a modified concept where concession of operating rights is granted for an existing facility and the return of control is not addressed.

investor take on some of the project specific risk. It is sometimes assumed that the choice of project financing versus internal capital just exists or can be explained entirely by financial factors such as risk spreading or shortage of internal capital financing.

Our paper will consider another factor that may influence the choice of project financing versus internal capital financing. The additional factor is the problem of management incentives for the parent company. Shifting risk of an investment project to the outside investor may discourage the appropriate level of effort by the parent company in managing the project. The resultant moral hazard problem leads to lower returns to the project. The general problem of the conflict between risk spreading and providing appropriate incentives to agents has a wide literature. As part of the literature there has been considerable discussion of the appropriate incentive structures to reward managers within a firm and whether to provide managerial rewards tied to firm profits or to provide fixed incomes. The issue of the appropriate incentive structure for rewarding managers within a firm is similar to the issue of project financing versus internal financing. However, analysis of project financing has received limited attention from this perspective.

There are differences between looking at incentive structures for managers and incentive structures in project financing. In analyzing managerial incomes, it may be a reasonable assumption that incomes are regulated by competitive forces which limit rewards to managers to normal returns for the work that they perform. Managers in general will have more limited opportunities to capture economic rents that firm owners may potentially earn due to unique knowledge, locational advantages or control of unique

inputs to productions. In contrast in this paper, the perspective will be that parent firms engaging in project financing may anticipate earning economic rent since competition to provide funds by outside investors sets a limit on the cost of project financing.

Risk sharing, managerial incentives and ownership patterns are at the heart of this paper. Although the business opportunity might be implemented under a range of intermediate arrangements, we consider only two extreme scenarios, namely a parent company acting as a principal supplying all capital investment versus acting solely as a management operator with all the investment capital supplied by outside investors.

We construct a model containing four important features. First, the project consists of the sale of the product on a stochastic market. Second, market outcomes are influenced by effort of management. Third, firms are risk averse. Lastly, risk neutral outside investors exist who will finance any project that generates non-negative expected value.<sup>3</sup> Based on these features a set of conditions are derived which determine whether the parent company uses internal capital to exploit a business opportunity or resorts to project financing. The paper is organized as follows: Section 2 introduces the basic framework of analysis. Section 3 presents the options available to the parent company. Results are derived in terms of when one option is preferred to the next and whether the parent company chooses to apply high versus

---

<sup>3</sup>By assumption we are including a normal rate of return for capital in project costs. A more complex model could consider circumstances where the outsider investor is not necessarily risk neutral but just less risk averse than the original parent company.

low managerial effort. Section 4 contains a few concluding remarks.

## II. MODEL

Consider a company who owns the franchise rights for a project.<sup>4</sup> The income of the firm is determined by revenue minus costs. Revenue depends on output,  $q$  and the inverse demand curve,  $P = a - bq$  for the firm's product.

$$(1) \quad R(q) = Pq = (a - bq)q$$

Total cost is given by:

$$(2) \quad C(q, e) = c_1 q^2 + c_2 q - c_3 e q + K + e$$

where

$e$  - effort is assumed to be one of two discrete levels either  $e_L$  or  $e_H$ , corresponding to low or high effort respectively,

$K > 0$  is the fixed capital investment in the project, and

To ensure the firm's cost is always an increasing function of output,  $c_2 > c_3 e_H$ . The parameter  $c_3$  reflects how much marginal and average cost are reduced per unit of effort. The project income,  $W$ , the difference between revenue and cost is influenced by a random variable.

$$(4) \quad W = u(R(q) - c(q, e)) \text{ where } u \text{ is a random variable.}$$

---

<sup>4</sup>Firms obtain specific advantageous business opportunities through acquiring unique knowledge or by the fortitious ownership of certain assets. The advantageous opportunity may be expected to generate economic rent but it is not a sure thing. Despite a considerable upside there may be substantial potential for failure.

The parameter  $u$  is a two-state random variable which takes the values  $u_2 > u_1$ . The state of the world,  $u$  influences  $W$  in a simple multiplicative fashion. This is an important simplifying assumption. In other applications one might suppose that uncertainty influences demand or perhaps cost in some way and then feeds through to uncertainty on profit. This adds undesirable complexity to the analysis and diverts attention away from focusing on the impact of risk. By assuming multiplicative uncertainty on corporate income, the output level will be chosen independently of the state of the world.

Without adding too much extra complication, it is assumed in the model that the probability of which state occurs is influenced by the level of effort undertaken by the management of the new business enterprise. For low effort,  $\text{prob}(u = u_1 * e_L) = \alpha$  and for high effort the  $\text{prob}(u = u_1 * e_H) = \psi \alpha$  where  $0 \neq \psi \neq 1$ . If  $\psi = 1$  then high effort has no impact on the riskiness of the project. At the other extreme,  $\psi = 0$ , high effort can eliminate the risk from the project. The low level of effort characterises an effort that can be contracted for and verified by an external investor. The high level of effort includes the low level of effort plus an effort level that can only be internally monitored by the parent company. The incremental effort cannot be contracted for and verified by external investors. When the parent firm works as an agent for external investors, the reward the parent firm receives is independent of the outcome in the market so there is no direct incentive to provide high effort unless the external investor contracts for it. The external investors do not contract for high effort because they can not determine whether it is being supplied or not.

Conventionally, effort might be put into many categories such as effort which is directed towards

reducing risk or effort which is directed towards cost reduction. Rather than distinguish between types we will assume, in our analysis that effort reflects general managerial competence and attentiveness. In this role, effort is not necessarily directed towards a specific management role. Low level effort corresponds to putting someone who meets the basic credentials of managing the project, a proverbial warm bum in the managerial seat of the new enterprise. A high level of corporate effort might involve putting a more skilled manager in charge of the new project and/or exercising some higher level monitoring of the quality of managerial activity. Higher levels of corporate effort involves costs in taking skilled managers from other areas and using up valuable senior management time. The previously cited activities involving the parent firm in higher effort levels are difficult for an external investor to verify or contract for.

The parent firm's utility function is assumed to exhibit Arrow-Pratt constant relative risk aversion with  $R$ , the measure of the Arrow-Pratt relative risk aversion. The parent company is assumed to be risk averse,  $0 < R < 1$ , maximizing the expected utility of corporate income,  $W$  from the new investment project. The utility of income is:

$$(3) \quad U(W) = W^{1-R}$$



### III. PROJECT FINANCING OPTIONS

Two different forms of project financing and compensation are considered.

The first is self-financing by the firm. The income of the firm will vary depending on the outcome in the market. The second alternative is for the parent company to sell the business opportunity and its management services to an outside investor in return for a fixed reward that is independent of the market outcome. Unfortunately in this case, since the high level of effort cannot be contracted for, the parent company will only supply low effort. However, in some cases, the parent company receives a reduction in risk that more than compensates for the lower average income generated by the project.

#### Case 1: Parent company self-financing

For a low level of effort the expected utility of profit is

$$EU(W|e_L) = \alpha u_1^{1-R} ((a-bq)*q - (c_1q^2 + c_2q - c_3 e_L q + K) - e_L)^{1-R} + (1-\alpha)u_2^{1-R} ((a-bq)*q - (c_1q^2 + c_2q - c_3 e_L q + K) - e_L)^{1-R}$$

correspondingly a high level of effort yields.

$$EU(W|e_H) = \psi\alpha u_1^{1-R} ((a-bq)*q - (c_1q^2 + c_2q - c_3 e_H q + K) - e_H)^{1-R} + (1-\psi\alpha)u_2^{1-R} ((a-bq)*q - (c_1q^2 + c_2q - c_3 e_H q + K) - e_H)^{1-R}$$

The firm must choose both effort and output before the state of the world is known

To choose the optimal level of effort the firm must calculate the maximum expected utility under the high effort level and under a low level of effort and compare the two.

For low effort,

$$\max EU(W|e_L) = \alpha u_1^{1-R} ((a-bq)^*q - (c_1q^2 + c_2q - c_3 e_L q + K) - e_L)^{1-R} + (1-\alpha)u_2^{1-R} ((a-bq)^*q - (c_1q^2 + c_2q - c_3 e_L q + K) - e_L)^{1-R}$$

As observed earlier, the profit of a firm is unaffected by whether the firm chooses output before or after the random variable  $u$  is observed. Since the state of world does not affect the choice, no distinction needs to be made between ex ante and ex post decisions on output.

The resultant optimal output is  $q^* = \frac{a-c_2+c_3e_L}{2(b+c_1)}$

Substituting for optimal output,  $q^*$  in the previous equations yields

$$EU^*(W|e_L) = (\alpha u_1^{1-R} + (1-\alpha)u_2^{1-R}) * ((a - c_2 + c_3 e_L)^2 / (4(b+c_1) - (K+e_L)))^{1-R}$$

to simplify our analysis, it is convenient to replace  $a - c_2 = B$  and  $b+c_1 = A$

$$EU^*(W|e_L) = (\alpha u_1^{1-R} + (1-\alpha)u_2^{1-R}) * ((B + c_3 e_L)^2 / (4A) - (K+e_L))^{1-R}$$

high effort will have an optimal output and expected utility similar in form to low effort

$$q^* = \frac{a-c_2+c_3e_H}{2(b+c_1)} = \frac{B+c_3e_H}{2A}$$

$$EU^*(W|e_H) = (\psi\alpha u_1^{1-R} + (1-\psi\alpha)u_2^{1-R}) * ((B+c_3e_H)^2 / (4A) - (K+e_H))^{1-R}$$

Under some specific simple conditions, it can be shown unambiguously that the firm will chose a high level of effort or a low level of effort. These are demonstrated in theorems 1 and 2 below.

**Theorem 1:** A firm will always chose a high level of effort if  $e_L \geq (2A - Bc_3) / c_3^2$ . Otherwise, a high level of effort may be desirable but it will depend on more complicated conditions

Proof: Only two levels of effort are actually available but one can consider the hypothetical impact of

other effort levels on expected utility.

$$EU(W|e) = (\psi\alpha u_1^{1-R} + (1-\psi\alpha)u_2^{1-R}) * ((B+c_3e)^2 / (4A) - (K+e))^{1-R}$$

the part that includes effort that influences utility is

$$T1(e) = (B+c_3e)^2 / (4A) - e$$

$$dT1/de = (Bc_3 + c_3^2e) / (2A) - 1$$

$$d^2T1/d^2e = c_3 / (2A) > 0$$

Since  $d^2T1/d^2e > 0$  solving for  $e^*$  where  $dT1/de = 0$ , yields a minimum expected utility.

$$e^* = (2A - Bc_3) / c_3^2$$

If  $e_L \exists e^*$  then  $dT1/de \exists 0$ , since  $d^2T1/d^2e > 0$  then  $dT1(e_H)/de > 0$ , high level of effort increases the expected utility above that of low level of effort.

Corollary: If  $2A - Bc_3 < 0$  then high effort will always be preferred

Proof:  $e_L > 0$  so the conditions of theorem 1 are always satisfied.

The above circumstance is favoured by  $A$  being small or  $Bc_3$  being large.

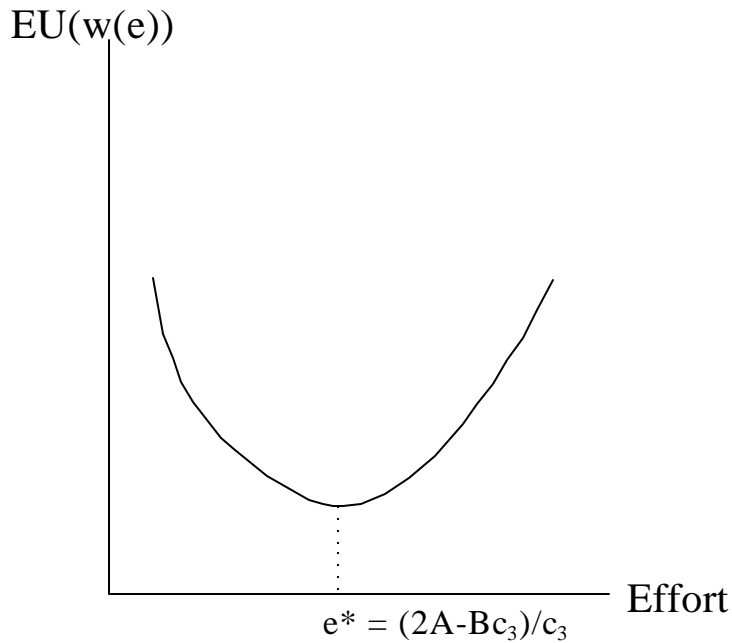
The term  $A = b + c_1$  is small if the market is large ( $b$  small) or diseconomies of scale,  $c_1$  is small. The term  $Bc_3$  is large if the maximum willingness to pay,  $a$  is large, or the unit variable cost,  $c_2$  is small or the impact of effort on reducing unit variable cost,  $c_3$  is large.

**Theorem 2:** If high effort has no impact on the likelihood of high or low income,  $\psi=1$  then low effort will yield higher expected utility whenever  $e_H \neq (2A - Bc_3) / c_3^2$ .

Proof: From theorem 1, since  $d^2T1/d^2e > 0$ ,  $dT1(e_H)/de \neq 0$  if  $e_H \neq (2A - Bc_3) / c_3^2$ . The expected

utility is increased as effort is reduced thus  $EU(e_H) < EU(e_L)$ . The condition,  $\psi=1$  is necessary because otherwise, higher effort increases the likelihood increasing the probability of favourable outcomes may compensate for the negative impact that high effort has on income in a given state of the world.

The relationship between effort and expected utility can be represented by the following diagram:



Sufficiently high effort if available, ultimately leads to greater income than lower effort because there are no diminishing returns to effort in this paper's model . Each additional amount of effort contributes a constant reduction in marginal and average cost. The reduction in cost encourages greater output which magnifies the rewards of greater effort. For all possible effort levels, constant improvement in average cost for effort is unrealistic. However, by assumption, our analysis will focus on considering only two discrete effort levels whose relevant range is characterized by effort making a constant improvement in

average cost due to additional effort.

## Case 2: Project financing by outside investors and fixed management fees

The principal has an alternative to financing the project and accepting its variable return. The principal can contract out the project to a risk neutral external investor who pays a fixed fee to the principal to manage the project. The fixed fee will include economic rent that the principal would expect to receive if the company had financed the project itself. The market for external investors is assumed to be competitive and the expected return of external investors need only be  $K$ , the cost of capital. Since the investors are making a fixed payment to the principal they assume all the risk. For the poor outcome, the investors will earn less than  $K$  and to compensate they will earn more than  $K$  when the outcome is most favourable.

As argued earlier, the investor cannot monitor whether the principal puts in high effort or not so that the payment the investor makes must be based on the expectation that the principal will provide low effort.

$EW$  will be the payment received by the parent company independently of the state of the world, net of the cost of the parent company's effort

$$EW = (\alpha u_1 + (1-\alpha)u_2) * ((B + c_3 e_L)^2 / (4A) - (K + e_L))$$

$$\begin{aligned} U(EW, e_L) &= ((\alpha u_1 + (1-\alpha)u_2) * ((B + c_3 e_L)^2 / (4A) - (K + e_L)))^{1-R} \\ &= (\alpha u_1 + (1-\alpha)u_2)^{1-R} * ((B + c_3 e_L)^2 / (4A) - C_L)^{1-R} \end{aligned}$$

Less expected wealth is created when the project is financed by an independent investor but the principal avoids risk and has a more certain return

Lemma 1: The principal prefers project financing if

$$G(\alpha, R, \psi, u_2) = \frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} > H(a, c_2, c_3, A, K, e_H) = \frac{(B^2 + 2Bc_3e_H + c_3^2e_H^2 - 4AK - 4Ae_H)^{1-R}}{(B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}}$$

the above is based on normalizing the functions by setting  $u_1 = 1$  and  $e_L = 1$ .

Proof: By definition, the principal prefers seeking project financing to providing its own funds if

$U(EW|e_L) > EU(W|e_H) > 0$  substituting the parameterized functions for these two relationships yields the requirement

$$(\alpha u_1 + (1-\alpha)u_2)^{1-R} ((B + c_3 e_L)^2 / (4A) - K + e_L)^{1-R} > (\psi\alpha u_1^{1-R} + (1-\psi\alpha)u_2^{1-R}) * ((B + c_3 e_H)^2 / (4A) - (K + e_H))^{1-R}$$

re-arranging the above equation and setting  $e_L = 1$  and  $u_1 = 1$  yields

$$\frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} > \frac{(B^2 + 2Bc_3e_H + c_3^2e_H^2 - 4AK - 4Ae_H)^{1-R}}{(B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}}$$

Lemma 2: The principal prefers project financing if he chooses low effort as the optimal action when self-financing.

Proof: The net value created is the same whether project financing or self-financing are undertaken at a low effort level. The difference is that the external investor absorbs all the risk. Comparing the expected profit of project financing and self-financing yields the following claim

$$\alpha + (1-\alpha)u_2^{1-R} (B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R} > (\alpha + (1-\alpha)u_2)^{1-R} (B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}$$

$$\alpha + (1-\alpha)u_2^{1-R} > (\alpha + (1-\alpha)u_2)^{1-R}$$

take both sides to  $1/(1-R)$  power:  $\alpha^{1/(1-R)} + \dots$  positive terms  $+ (1-\alpha)u_2 > \alpha + (1-\alpha)u_2$

$\alpha^{1/(1-R)} + \dots$  positive terms  $> \alpha$  which must be true since  $0 < R < 1$ .

**Lemma 3:** If  $e_L \exists (2A - Bc_3)/c_3^2$ , the principal will only choose project financing if

$$\frac{(\alpha + (1 - \alpha)u_2)^{1-R}}{(\psi\alpha + (1 - \psi\alpha)u_2^{1-R})} > 1$$

and the above term will always be less than  $u_2$ .

Proof: See appendix

Lemma 1 and 3 imply certain restrictions on situations where a firm will chose project financing.

**Theorem 3:** For  $e_L \exists (2A - Bc_3)/c_3^2$ , if  $G(\alpha, R, \psi, u_2) = \frac{(\alpha + (1 - \alpha)u_2)^{1-R}}{(\psi\alpha + (1 - \psi\alpha)u_2^{1-R})} > 1$  then a critical  $e_H^*$

exists such that for all levels  $e_H < e_H^*$  project financing will be favoured.

Proof: Let  $G(\alpha, R, \psi, u_2) = 1 + \gamma$  with  $\gamma > 0$ , project financing is preferred for all  $e_H$

that satisfy  $H(a, c_2, c_3, A, K, e_H) = \frac{(B^2 + 2Bc_3e_H + c_3^2e_H^2 - 4AK - 4Ae_H)^{1-R}}{(B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}} \# 1 + \gamma$

From lemma 3,  $1 + \gamma < u_2$  so a finite value exists for H. If  $e_H = e_L$  then the above must be

satisfied since the right hand side will be equal to 1. From Theorem 1,  $dEU(W, e)/de > 0$  so that

as  $e_H$  is increased, the numerator increases so that an  $e_H = e_L + \lambda$  exists where

$$H(a, c_2, c_3, A, K, e_L + \lambda) = 1 + \gamma.$$

Under the conditions specified by Theorem 3, it is possible to specify some comparative statics

implications about how different factors will influence whether a firm chooses internal financing or uses

project financing.



**Theorem 4:** If  $e_L \exists (2A - Bc_3)/c_3^2$  and  $G(\alpha, R, \psi, u_2) = \frac{(\alpha + (1 - \alpha)u_2)^{1-R}}{(\psi\alpha + (1 - \psi\alpha)u_2^{1-R})} > 1$

then outside investor financing is favoured by:

- i) increasing  $\psi$ , the smaller the impact effort has on increasing the likelihood of favourable outcomes.
- ii) decreasing  $u_2$ , the smaller the favourable outcome
- iii) increasing B, if  $B < \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2 e_H^2 - 8e_H A - 2c_3^2 e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$   
and decreasing B, if  $B > \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2 e_H^2 - 8e_H A - 2c_3^2 e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$
- iv) increasing A, if  $B > c_3(K + (K^2 + e_h K + e_h + K)^{1/2})$  and  
decreasing A if  $B < c_3(K + (K^2 + e_h K + e_h + K)^{1/2})$  where increasing A represents decreasing market size, b or larger diseconomies of scale,  $c_1$
- v) decreasing  $c_3$ , if  $c_3 > \frac{((1 + e_h)(4AK - B^2) + 4A e_h + (16(e_h^2 K^2 + 2e_h K + K^2 + e_h^2 + 2 e_h^2 K - 2e_h K^2)A^2 + (e_h^2 - 2e_h + 1)B^4 - 8A(e_h + e_h^2 K + K + e_h^2))B^2)^{1/2}}{(2e_h B)}$   
increasing  $c_3$ , if  $c_3 < \frac{((1 + e_h)(4AK - B^2) + 4A e_h + (16(e_h^2 K^2 + 2e_h K + K^2 + e_h^2 + 2 e_h^2 K - 2e_h K^2)A^2 + (e_h^2 - 2e_h + 1)B^4 - 8A(e_h + e_h^2 K + K + e_h^2))B^2)^{1/2}}{(2e_h B)}$   
where increasing  $c_3$  represents an increasing reduction in average cost per unit of effort.
- vi) increasing K, the cost of investment in the project
- vii) decreasing  $e_h$ , the high level of effort

Proof: See appendix.

Theorem 4 suggests a range of factors that will tend to push firms towards or away from project financing. Factors such as how much additional effort better management can contribute, the value of additional effort in reducing risk, the size of fixed capital, and the upside potential on income have unambiguous effects. In contrast, factors such as average variable costs, average cost impact of effort and size of the market are more complicated in their impact.

The smaller the contribution of unobservable effort in determining expected profits the more likely a firm will choose project financing. The impact of unobservable effort is reduced by :

- i) increasing  $\psi$ , which implies that effort is less important in determining favourable outcomes
- ii) decreasing  $u_2$ , the smaller the favourable outcome and
- iii) decreasing  $e_h$ , the high level of effort available above the effort which can be contracted for.

Another factor that increases the likelihood of project financing is increased  $K$ , the fixed capital investment required. As  $K$  is increased other factors constant the riskiness of the project is increased with no compensating benefits. Higher risk with no additional benefits increases the relative advantage of employing project financing to avoid risk.

The impact of increasing market size on the decision on whether to use project financing depends on the interaction of two factors fixed investment,  $K$  and the maximum surplus of the market,  $B=a-c_2$ .

Maximum surplus of the market reflects the maximum incremental value that can be gained from producing one unit of output. The maximum surplus of the market provides a measure of the potential gains from trade available for producing the product. The relationship between maximum surplus of the market and fixed investment costs,  $K$  determine whether outside investment is favoured by larger or smaller markets. If fixed investment requirements are large relative to the maximum surplus of the market then project financing will tend to be favoured when markets are small. On the other hand if fixed investment is small relative to the maximum surplus of the market then increasing size of the market

will favour outside investment. The relationship between fixed investment costs and maximum surplus is an indicator of how the relationship between risk and reward will be influenced by increases in market size. In the case, where fixed capital costs are large and potential market surplus is small, relative risk will increase as market size gets smaller. Conversely when fixed capital costs are small and potential market surplus is large the relative riskiness of profits increases as market size becomes larger.

For a given set of other factors, a specific level  $B^*$  exists for  $B$ , the maximum consumer surplus, which most favours project financing. Project financing is less favoured as  $B$  becomes increasingly larger or smaller than  $B^*$ .

Similarly for the impact of effort on unit average cost,  $c$ , there is a critical  $c^*$  where the relative benefit from project financing is at a maximum. The benefit diminishes as  $c$  becomes larger than  $c^*$  and also diminishes as  $c$  becomes smaller than  $c^*$ .

Theorem 4 is focused on circumstances where the choice is at the borderline between outside and internal financing. The results do not indicate over a broad range whether profits from one type of financing or another are increased or decreased by a particular parameter. Theorem 4 indicates how the utility of profits for project financing is affected relative to internal funding of capital projects.

The critical point is where

$$G(.*) = \frac{Gn(.*)}{Gd(.*)} = \frac{Hn(.*)}{Hd(.*)} = H(.*)$$

$$G_n(.*)H_d(.*) = G_d(.*)H_n(.*)$$

utility of outside financing

utility of internal financing

if either G increases or H decreases then outside financing is favoured

However, if  $G_n(.*)H_d(.*) - G_d(.*)H_n(.*) > 0$  (gain in utility from outside)

and either G increases or H decreases alone it is not necessarily true that utility from outside investment

rises.

## V. CONCLUSION

The model highlights the conditions under which a particular ownership structure is best suited for carrying out a capital project. The choice to use project financing is affected by two major factors, namely the extent and nature of the uncertainty and the required amount of investment.

The results derived for the parent firm arranging for outside financing are similar to the standard principal-agent relationship,<sup>5</sup> there are, however, two main differences. First, the assumption of a large number of potential investors implies that all the negotiating power goes to the firm in question and the outside investor only earns a competitive rate of return. Second, the on-going relationship with the parent company suggests the firm must have strategic considerations in its involvement. That is the parent company cannot or prefers not to sell its idea or knowledge to the investors and have nothing to with the management of the new enterprise.

A range of factors influence the decision of a firm to choose internal financing versus project financing. Firms will tend to prefer internal financing of investment when effort has a significant impact on the magnitude and likelihood of favourable outcomes. Conversely, other factors equal the larger the capital investment the more likely outside financing will be favoured. The impact of the size of market will depend on how large fixed capital investment is relative to the maximum surplus (the difference between maximum willingness to pay and average variable cost) available from production. If fixed capital costs

---

<sup>5</sup>The discussion here ignores the possibility of selling the idea of the project for money. This assumption simplifies the firm's willingness to exert effort for a given contractual earnings.

are large relative to the maximum surplus then project financing is favoured in smaller markets. Conversely if fixed capital costs are small relative to the maximum surplus available then project financing is favoured by larger markets. Lastly, for factors such as maximum consumer surplus,  $B$ , and contribution of effort to reducing average cost,  $c$ , project financing will be favoured in an intermediate range of potential circumstances for businesses operations.

The analysis in this paper has focused on two extreme cases: all internal financing versus complete project financing by outside investors. An interesting area of future research would be to consider an intermediate case where the parent company shares the financing with an outside investor each putting up part of the investment funds. This arrangement would attempt to balance benefits in risk reductions with incentives for managerial effort by the parent firm.

## Appendix:

Longer proofs

### Lemma 3:

Trivially the low effort case must earn positive profits if it is worth considering. Given this is the case, from Theorem 1 since  $e_H > e_L$  then  $dEU(W, e_H)/de > 0$  which implies

$$(B^2 + Bc_3 e_H + c_3^2 e_H^2 - 4AK - 4Ae_H)^{1-R} > (B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}$$

it follows from Lemma 1 that

$$\frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} > 1$$

the second claim

$$\frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} < u_2$$

$$\frac{\max((\alpha + (1-\alpha)u_2)^{1-R})}{\min((\psi\alpha + (1-\psi\alpha)u_2^{1-R}))} = u_2$$

since  $\max((\alpha + (1-\alpha)u_2)^{1-R}) = u_2$  with  $\alpha = 0, R=1$  and  $\min((\psi\alpha + (1-\psi\alpha)u_2^{1-R})) = 1$  with  $\alpha = 1, \psi=1$

furthermore since the numerator cannot be at a maximum while the denominator is at a minimum

$$\frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} < u_2$$

**Theorem 4:** If  $e_L \exists (2A - Bc_3)/c_3^2$  and  $G(\alpha, R, \psi, u_2) = \frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2^{1-R})} > 1$

then outside investor financing is favoured by:

i) increasing  $\psi$ , the smaller the impact effort has on increasing the likelihood

of favourable outcomes.

ii) decreasing  $u_2$ , the smaller the favourable outcome

iii) increasing B, if  $B < \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2 e_H^2 - 8e_H A - 2c_3^2 e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$

and decreasing B, if  $B > \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2 e_H^2 - 8e_H A - 2c_3^2 e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$

iv) increasing A, if  $B > c_3(K + (K^2 + e_h K + e_h + K)^{1/2})$  and

decreasing A if  $B < c_3(K + (K^2 + e_h K + e_h + K)^{1/2})$  where increasing A represents decreasing market size, b or larger diseconomies of scale,  $c_1$

v) decreasing  $c_3$ , if  $c_3 > \frac{((1 + e_h)(4AK - B^2) + 4A e_h + (16(e_h^2 K^2 + 2e_h K + K^2 + e_h^2 + 2 e_h^2 K - 2e_h K^2)A^2 + (e_h^2 - 2e_h + 1)B^4 - 8A(e_h + e_h^2 K + K + e_h^2))B^2)^{1/2}}{(2e_h B)}$

increasing  $c_3$ , if  $c_3 > \frac{((1 + e_h)(4AK - B^2) + 4A e_h + (16(e_h^2 K^2 + 2e_h K + K^2 + e_h^2 + 2 e_h^2 K - 2e_h K^2)A^2 + (e_h^2 - 2e_h + 1)B^4 - 8A(e_h + e_h^2 K + K + e_h^2))B^2)^{1/2}}{(2e_h B)}$

where increasing  $c_3$  represents an increasing reduction in average cost per unit of effort.

vi) increasing K, the cost of investment in the project

vii) decreasing  $e_h$ , the high level of effort

Proof: Suppose that  $e_H = e_H^*$ , so the firm is just indifferent between investing itself and using outside investors. From Theorem 3 such an effort level must exist. This implies



$$G(\alpha, R, \psi, u_2) = \frac{(\alpha + (1-\alpha)u_2)^{1-R}}{(\psi\alpha + (1-\psi\alpha)u_2)^{1-R}} = H(A, B, c_3, A, K, e_H) = \frac{(B^2 + 2Bc_3e_H + c_3^2e_H^2 - 4AK - 4Ae_H)^{1-R}}{(B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^{1-R}}$$

i) G(.) always gets larger when  $\psi$  increases since

$$dG(\alpha, R, \psi, u_2)/d\psi = \frac{(\alpha + (1-\alpha)u_2)^{1-R} (\alpha u_2^{1-R} - \alpha)}{((\psi\alpha + (1-\psi\alpha)u_2)^{1-R})^2} > 0 \quad \text{since } u_2 > u_1 = 1, \alpha \neq 1 \text{ and } 0 < R < 1$$

H(.) remains constant since  $dH/d\psi = 0$ . This favours using outside investors.

ii) G(.) always gets smaller when  $u_2$  increases since

$$dG(\alpha, R, \psi, u_2)/du_2 = \frac{\alpha u_2 (\psi(1-\alpha) - (1-\psi\alpha)u_2^{1-R}) (\alpha + (1-\psi\alpha)u_2)^{1-R} (1-R)}{((\psi\alpha + (1-\psi\alpha)u_2)^{1-R})^2 (\alpha + (1-\alpha)u_2)u_2} < 0$$

since  $\psi \neq 1, u_2 > u_1 = 1, \alpha \neq 1$  and  $0 < R < 1$

H(.) remains constant since  $dH/d\psi = 0$ . Increasing  $u_2$  will favour using outside investors.

iii) To prove claim 3 one must determine how H(.) changes when B increases. To do this it is useful to define a function,  $T2(.) = H(.)^{(1/(1-R))}$ . The sign of  $dH/dB$  is preserved for the function T2(.) since  $H(.) > 1$

$$T2(.) = \frac{(B^2 + 2Bc_3e_H + c_3^2e_H^2 - 4AK - 4Ae_H)}{B^2 + 2Bc_3 + c_3^2 - 4AK - 4A}$$

$$dT2./dB = - \frac{2(e_H - 1)(c_3^3e_H + Bc_3^2(e_H + 1) + B^2c_3 + 4AKc_3 - 4AB)}{(B^2 + 2Bc_3 + c_3^2 - 4AK - 4A)^2}$$

the following condition signs the above derivative:

$$\text{If } B > \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2e_H^2 - 8e_HA - 2c_3^2e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$$

then  $dT2./dB < 0$

G(.) remains constant since  $dG/dB = 0$ . The term  $dT(.) / dB < 0$  implies  $dH(.) / dB < 0$ . Increasing B favours using outside investors since the relative utility to using outside investor is increased. Since  $B = a - c_2$ , B is larger with increases in the maximum willingness to pay, a or decreases in the unit variable cost

of output,  $c_2$ .

$$\text{otherwise if } B < \frac{4A - c_3^2(e_H + 1) + (c_3^2(c_3^2 e_H^2 - 8e_H A - 2c_3^2 e_H - 8A + c_3^2 - 16AK) + 16A^2)^{1/2}}{2c_3}$$

then  $dT_2(\cdot)/dB > 0$

$G(\cdot)$  remains constant since  $dG/dB = 0$ . The term  $dT(\cdot)/dB > 0$  implies  $dH(\cdot)/dB > 0$ . Decreasing  $B$  favours using outside investors since the relative utility to using outside investor is increased. Since  $B = a - c_2$ ,  $B$  is larger with increases in the maximum willingness to pay,  $a$  or decreases in the unit variable cost of output,  $c_2$ .

iv) Again use the  $T_2(\cdot)$  function to determine the impact on  $H(\cdot)$ .

Taking the derivative of  $T_2(\cdot)$  with respect to  $A$ .

$$\frac{dT_2(\cdot)/dA}{(B^2 + Bc_3 + c_3^2 - 4AK - 4A)^2} = \frac{4(e_H - 1)(e_H K + e_H + K)c_3^2 + 2KBc_3 - B^2}{(B^2 + Bc_3 + c_3^2 - 4AK - 4A)^2}$$

Two cases exist if:

$$\text{a) } B > c_3 (K + (K^2 + e_h K + e_h + K)^{1/2})$$

then  $dT_2(\cdot)/dA < 0$  and  $dH/dA < 0$ , since  $dG/dA = 0$ , an increase in  $A$  increases the possibility of a firm choosing to use an outside agent, otherwise if

$$\text{b) } B < c_3 (K + (K^2 + e_h K + e_h + K)^{1/2})$$

then  $dT_2(\cdot)/dA > 0$  and  $dH/dA > 0$ , since  $dG/dA = 0$ , a decrease in  $A$  increases the possibility of a firm choosing to use an outside agent.

v) The impact of  $c_3$  can again be traced out through its impact on  $T_2(\cdot)$

$$\frac{dT_2(\cdot)/dc_3}{(4A(K+1) - (B+c_3)^2)^2} = \frac{-2(1 - e_h)(c_3^2 e_h B - c_3(e_h(4AK + 4A - B^2) + 4AK - B^2) + B^3 - 4BAK)}{(4A(K+1) - (B+c_3)^2)^2}$$

the following condition signs the above derivative:

$$\text{If } c_3 > ((1 + e_h)(4AK - B^2) + 4A e_h + (16(e_h^2 K^2 + 2e_h K + K^2 + e_h^2 + 2 e_h^2 K - 2e_h K^2)A^2$$

$$+ (e_h^2 - 2e_h + 1)B^4 - 8A(e_h + e_h^2 K + K + e_h^2))B^2)^{1/2} / (2e_h B)$$

then  $dT2(\cdot)/dc_3 > 0$

$G(\cdot)$  remains constant since  $dG/dc_3 = 0$ . The term  $dT(\cdot)/dc_3 > 0$  implies  $dH(\cdot)/dc_3 > 0$ . Decreasing  $c_3$  favours using outside investors since the relative utility to using outside investor is increased. Decreases in  $c_3$ , lower the reduction in cost per unit of effort  $a$ . In this case, there is less of penalty for low levels of effort.

otherwise if  $c_3 < ((1+e_h)(4AK-B^2)+4A e_h+(16(e_h^2 K^2 +2e_hK+K^2+e_h^2+2 e_h^2K-2e_hK^2)A^2 + (e_h^2-2e_h+1)B^4-8A(e_h+e_h^2K+K+e_h^2))B^2)^{1/2}/(2e_hB)$

then  $dT2(\cdot)/dc_3 < 0$

$G(\cdot)$  remains constant since  $dG/dc_3 = 0$ . The term  $dT(\cdot)/dc_3 < 0$  implies  $dH(\cdot)/dc_3 < 0$ .

In this case increasing  $c_3$ , reduction in cost per unit of effort favours outside investment. The reason is quite subtle. The  $Aprofit \equiv$  for high effort is increased more than for low effort but in relative terms low effort greater benefits more in this case which is indicated by a decline in  $H(\cdot)$

vi) Proving claim vi), as with iv) and v) it is useful to focus on how  $T2(\cdot)$  is affected by changes in  $K$ .

Differentiating  $T2(\cdot)$  with respect to  $K$

$$\frac{dT2(\cdot)/dK}{(B^2+2Bc_3+c_3^2-4AK-4A)^2} = \frac{4A((B^2+2Bc_3+c_3^2-4AK-4A)^{-1} - (B^2+Bc_3 e_H+c_3^2 e_H^2-4AK-4Ae_H))}{(B^2+2Bc_3+c_3^2-4AK-4A)^2}$$

by assumption,  $e_L \exists (2A-Bc_3)/c_3$ , from Theorem 1

$$(B^2+Bc_3 e_H+c_3^2 e_H^2-4AK-4Ae_H)^{1-R} > (B^2+2Bc_3+c_3^2-4AK-4A)^{1-R}$$

since both terms are positive this implies

$$B^2+Bc_3 e_H+c_3^2 e_H^2-4AK-4Ae_H > B^2+2Bc_3+c_3^2-4AK-4A \text{ so}$$

$$dT2(\cdot)/dK < 0$$

G(.) remains constant since  $dG/dK = 0$ . The term  $dT(\cdot)/dK_3 < 0$  implies  $dH(\cdot)/dK_3 < 0$ . Over this range increasing K, capital favours using outside investors. High capital costs raise the risk to self-financing and increase the relative benefits of using outside investors.

vii) Differentiating T2(.) with respect to  $e_H$

$$\frac{dT2}{de_H} = \frac{2Bc_3 + 2e_H c_3^2 - 4A}{B^2 + 2Bc_3 + c_3^2 - 4AK - 4A} > 0$$

since the denominator, the Utility of low effort is positive by assumption and the numerator is positive because  $e_H > e_L \exists (2A - Bc_3)/c_3^2$ . Since  $dT2/de_H > 0$  this implies  $dH/de_H > 0$ , as  $e_H$ , the high effort possible, is reduced, it lowers the benefits of self-financing and increases the relative benefit of using outside financing.

## REFERENCES

1. Brealey, R., S. Myers, G. Sick and R. Giammarino, Principles of Corporate Finance, 2<sup>nd</sup> Canadian Edition, McGraw-Hill, 1992.
2. Brealey, R.A., I.A. Cooper and M.A. Habib, AUsing Project Finance to Fund Infrastructure Investments,≡ Journal of Applied Corporate Finance, Fall 1996.
3. Cenatempo, D.J., AProject Financing Offers Investment Option with Low Borrower Risk,≡ Pulp and Paper, Vol. 70, Dec. 1996, pp 93-98.
4. Chehayl, P. and E.E. Berger, ASprint to the Finish on Financing Projects,≡ Financial Executive, Vol. 10, Jan./Feb. 1994, pp 17-21.
5. De Fraja, G., AEntrepreneur or Manager: Who Runs the Firm?≡ Journal of Industrial Economics, Vol. 1, March 1996, pp 89-98.
6. Kensinger, J.W. and J.D. Martin, AProject Financing: Raising Money the Old-fashioned Way,≡ Midland Corporate Finance Journal, Fall 1998.
7. Milgrom, P., and J. Roberts, Economics, Organization and Management, Prentice-Hall, 1992.
8. Ross, H., ABuilding on the Benefits of Project Financing,≡ Financial Times Mastering Finance Series, Nov. 1997, pp 80-85.
9. Thomadakis, S., and N. Usmen, AForeign Project Financing in Segmented Capital Markets: Equity Versus Debt,≡ Financial Management, Winter 1991, pp 42-53.
10. Tirole, J., The Theory of Industrial Organization, MIT Press, 1988.