A Structural Model of Pension Program Choice[1]

Kristin J. Kleinjans
University of Pittsburgh
Department of Economics
E-mail: kjkst44@pitt.edu

December 2002

JEL codes: H55; G23; O12

Keywords: structural model, structural estimation, pension choice, political risk, pension reform, Colombia

Abstract

Many countries have reformed their pension programs in the past years, or have at least discussed solutions to the financial imbalances caused by the demographic transition. Little is known, however, about the preferences of the insured for different kinds of pension programs. I develop a structural model of pension program choice, explicitly modeling the impact of the expected political risk of a pension reform and the resulting expected cuts in benefits. Using unique individual data from Colombia, where the insured have the choice between a publicly administered pay-as-you-go program and a privately administered fully funded program, I estimate the underlying utility parameters and use those to predict switching behavior following benefit reduction in the public program. I find that most of the choices can be explained as the outcome of pension income maximization, and that insured choose systematically differently depending on their level of education, age at a given educational level, and income. A simulation of the impact of a proposed pension reform suggests that surprisingly few of the insured are expected to switch as a result of benefit cuts and increases in retirement ages in the public program.

---

[1] Financial support from the Center for Latin American Studies at the University of Pittsburgh and the William and Flora Hewlett Foundation is gratefully acknowledged. I would like to thank Norberto Rojas and Alejandro Mateus Amaya from the National Planning Department in Colombia for providing access to the data. Special thanks to Daniel Berkowitz and Martin Gaynor who provided many helpful suggestions and valuable advice. I would also like to thank Andrew Samwick, Patricia Beeson, and Siddharth Chandra for their very helpful comments. All remaining errors are my own.

1 Introduction

Pension reforms that introduce some elements of choice between different types of pension programs have been very popular in the last decade as a way to ease the financial imbalances of the public pension programs caused by the demographic transition. Examples of countries that conducted such reforms are Argentina, Peru, and the United Kingdom. In the US, the voluntary or mandatory diversion of part of the pension insurance contribution to private, individual accounts has been seriously discussed both in the political and academic arena for several years.

Little is known, however, about the preferences of the insured for different kinds of pension programs. This paper aims to shed light on this question and, in particular, if the choice can be explained as the outcome of a maximization of expected pension income and how the insured would react to reforms within the public pension programs. I develop a structural model of pension program choice, explicitly modeling the expected political risk of a pension reform and the resulting expected cuts in benefits as an expected default on the rate of return in the public program. The insured maximize the expected rates of return as a measure of future expected income.

Using a unique data set from Colombia, a country in which the insured have the choice between a publicly administered pay-as-you-go program and a privately administered fully funded program, I estimate the underlying utility parameters and use those to predict switching behavior following reductions in benefits in the public program. I find that the model can explain the choice of the insured very well if I correct for heteroskedasticity in the level of education, age at a given educational level, and income, which are variables that systematically affect the choice behavior of the insured. I simulate the impact of a proposed pension reform by the Colombian government and find that surprisingly few of the insured are expected to switch as a result of benefit cuts and increases in the retirement ages in the public program. This suggests that changes in the public pension program will not significantly affect the decision of the insured. For those of the insured that are on the margin, an important variable in determining the reaction to policy changes is the change in the default expectation. Consequently, the

Colombian government should be able to improve the financial imbalance of the public program by cutting benefits without worsening its financial instabilities by inducing too many of the insured to switch from the public to the private program. This risk can be further reduced if the reform increases the confidence in the long-term stability of the public program.

The literature on pension reforms is vast. Several recent papers look at the impact of the introduction of choice in particular with respect to the switching behavior of the insured. However, most of these papers use aggregate data (such as Palacios and Whitehouse (1998) and Disney, Palacios and Whitehouse (1999)) or a reduced form analysis (such as Augusztinovics et al (2002: 54 – 58)). Augusztinovics et al (2000) analyze switching behavior in Hungary after its pension reform of 1998. Using a logistic regression model for the choice of switching, one of their major findings is that age and education have a significant effect on switching. Palacios and Whitehouse (1998) find a consistent and rational pattern of switching in a variety of countries, and suggest that this pattern of switching could be influenced through the design of the valuation of past contributions, the minimum pension guarantee and the reformed public program. Disney, Palacios and Whitehouse (1999) find that the vast majority of the insured responded "rationally" to switching incentives in the UK regarding the possibility of opting-out of the public SERPS program.[2]

Several authors conduct simulations of the impact of the introduction of choice between defined-benefit and defined-contribution programs. Brugiavini and Disney (1995) assume equal expected returns in both programs, investment risk in the defined-contribution plan and risk of reduced pension rights due to job turnover in the defined-

[2] A variety of recent papers investigate possible non-optimal behavior of insured and find evidence of status-quo bias (Samuelson and Zeckhauser 1988), the influence of the default option (Madrian and Shea 2001; Choi, Laibson, Madrian and Metrick 2001a and 2001b), and the use of rules of thumb (Benartzi and Thaler 2001) to choose between different pension programs or the allocation of retirement funds. Since the data used in this paper is cross sectional, I am not able to investigate these questions.

benefit program. Gustman and Steinmeier (1998) simulate first-round effects of the introduction of a voluntary choice between social security and a private program on program participation, the path of benefit and taxes, and retirement behavior. Kotlikoff, Smetters and Walliser (1998) compare two methods of privatizing social security (mandatory and voluntary) using a perfect-foresight heterogeneous agent overlapping generations model. Samwick (1998) uses data from the Survey of Consumer Finances to estimate the effects of the introduction of a voluntary buy-out of social security on national saving in the US.

None of these authors has data on actual individual choice available. In contrast, the data used in this paper allows me to develop a structural model of pension program choice in Colombia interpreting the individual choice between the pension programs as a revealed preference. This enables me to make predictions about the switching behavior of the insured in the case of changes in the pension system, such as a cut in benefits. Using a random utility model (McFadden 1981), I estimate the utility parameters given the uncertain rates of return in the public and the private program. The rate of return used for the public program is the internal rate of return (IRR) - the interest rate that equates the present value of contributions and the present value of benefits (Geanakoplos, Mitchell and Zeldes 1999). The market return is used as the return of the private program. The choice of the rate of return is chosen over alternative measures of expected future pension income, because it avoids the need for assumptions on the discount rate (as needed for present value comparison, for example) and makes an easy comparison with the return obtained in the private program possible.[3] An alternative but less transparent measure would be the accrual rate, which requires assumptions on annuity costs in the private program.[4]

---

[3] See Leimer (1995) for a discussion of money's worth measures in pay-as-you-go programs.

[4] Note that using the accrual rate would require to use the replacement rate since all past contributions are transferred to the other program (Disney, Palacios and Whitehouse 1999: 16).

Public and private pension programs suffer from different kinds of risk.[5] This paper includes the effects of two expected risks: the expected political risk that the state defaults on the pension promises in the public program (Feldstein 1997),[6] and the expected investment risk in the private program (with respect to uncertain return and volatility). The model abstracts from several benefits in the two programs, including longevity insurance in the public program (which can be bought at a cost in the form of an annuity in the private program), and the possibility of bequest and withdrawal of some funds if sufficient capital is accumulated in the private program. It also does not explicitly take into account the switching possibility. In the absence of political risk in the public program, the insured should always choose the private program, because it is always possible to switch to the public program if the realized states in the private program are less beneficial than the entitlement conditions in the public program. Political risk, however, introduces uncertainty about the continued existence of the switching possibility. Since an elimination of the switching possibility has been in the discussion for a variety of years due to the adverse selection issues arising from the switching possibility, in the following it is assumed that the insured do not expect to be able to switch in the future. I also assume that the political risk is restricted to the public program, in the sense that the affiliates of the private program do not expect their accumulated funds or pensions to be taxed to help finance the public program.

This paper is organized as follows. The next section describes the institutional setting and the data used in the analysis. In Section 3, the model is developed. The specification is explained in Section 4, and Section 5 shows the estimation and its results under various parameters specifications. The results of simulations of changes in the pension system that are currently in discussion in Colombia are shown in Section 6. The last section concludes.

---

[5] See Augusztinovics et al (2002: 53).

[6] See McHale (1999) for an analysis of the political risk of pension programs in the G-7 countries.

2 Institutional Setting and Data Source

2.1 Institutional Setting

Colombia reformed its pension system in 1994, introducing a private program that coexists with the public program. At the same time, contribution rates in the public program were increased and retirement benefits cut. Contribution rates are the same in both programs (for more detailed information on the Colombian pension system see Kleinjans 2003a), and both programs have the same kind of survivor and disability insurance, so that these aspects can be excluded from the analysis.

In the private program, the insured are allowed to retire as soon as the accumulated capital is sufficient to buy an annuity in the amount of 110% of the minimum wage. As of August 2002, there existed six different pension fund administrators that each offers one investment fund to the insured. The returns achieved by the SAFPs so far have been relatively similar, which is most probably due to a variety of investment regulation and the fact that the administrators must guarantee a relative minimum return (Srinivas and Yermo 1999; Turner and Rajnes 2001). The financial sector created a guarantee fund (FOGAFIN) that guarantees 100% of the contributions in case of the loss of a fund (Paredes 1997: 95), and a last resort state-guarantee if the guarantee fund is depleted or the mandatory reserves of the individual SAFP are not sufficient to guarantee the minimum return. This decreases the risk of fraud and bankruptcy of the SAFPs. The biggest uncertainty for the insured seems to be, therefore, about the amount and volatility of the real return.

The retirement age in the public program is 55 years for women and 60 years for men, and is supposed to be increased by two years each in 2014. The pension depends on the number of contributed weeks and the inflation-adjusted average wage of the last ten years of the work life. Monthly pensions are very generous and range between 65% and 85% of the above-mentioned average wage and are, in addition, paid 14 times a year. This in conjunction with low contribution rates leads to financial instability of the

program and likely future benefit cuts and increases in retirement ages (see Kleinjans 2003a for more information).

The internal rates of return of the public program are well above the steady state level. As Samuelson (1958) and Aaron (1966) have shown, the steady-state return of a pay-as-you-go system without reserves is equal to the sum of wage and population growth. Population growth in Colombia is around 1.8%, and wage growth averaged 1.9% in the 1970's and 1980's, and has since steadily declined to 0.9% in 1990-1993 (World Bank 2000 and ECLAC 1996: 93). If the wage growth stays somewhere between those numbers, the steady-state internal rate of return is between 2.7% and 3.7%. The internal rate of return in the public program ranges, however, from at least 4.4% to over 8% (Kleinjans 2002). This political risk that the state might default on its pension promises and conduct a pension reform that cut benefits and/ or increases retirement ages will in the following be called default risk.

The insured can choose between the two programs and switch every three years. If an insured switches from the public to the private program, a recognition bond is issued to account for past contributions. This recognition bond yields an annual real interest rate of 4% if the switch occurred before 1999, and 3% thereafter. If the insured switches from the private to the public program, the accumulated funds are transferred to the public program and the insured gets the contributed weeks at the respective wages credited.

During the transition to the new system, older insured and the insured that had contributed during a relatively long time into the old system were either exempted from the reform or could choose to retire in the old system under more beneficial conditions. In the following, these insured are excluded from the analysis.

2.2 Data Source

For the analysis, data from the Colombian National Survey About Life Quality from 1997, a national household survey, is used. The complete sample consists of observations for 38,518 individuals. Of those, 12,780 individuals reported to be working, out of which 7,708 work for a salary. Of those, 3,138 are affiliated with a pension

program, out of which 2,622 are affiliated with either the private or public program. The remaining individuals are affiliated with other pension programs, such as the one for the military forces. Excluding the insured that, because of their age, were excluded from choosing or can choose to retire to more beneficial pre-reform conditions as well as one outlier that had an extremely high salary, the remaining working sample encompasses 1,430 observations (see Table 1 for some of the characteristics of the insured in the sample).

Table 1: Some Characteristics of the Insured in the Data Set [a]

Variable	Explanation	Mean	Standard Deviation	Min/ Max
Public	=1 if in public program	0.61	0.49	0 / 1
Age	age in years	30.25	5.94	15/ 42
Salary	salary in thousand pesos	413,384	399,872	15 / 4,250 [b]
Female	= 1 if female	0.37	0.48	0 / 1
Educa	education level: = 1 if primary or less; = 2 if secondary; = 3 if tertiary	2.13	0.64	1 / 3

[a] For the working sample of 1430 insured.
[b] This corresponded to about $13/ $3,724 US-Dollars in 1997 (DANE 2002).

3 The Model

In the following, I develop a structural model of pension program choice. In this model, the insured choose the pension program that gives the highest expected utility, which depends on the expected rate of return as well as its variance. I can empirically estimate this model because the data used, as described in the last section, includes information on the individual pension program choice in Colombia. A special feature of the model is the introduction of an expected risk of default on the pension promises as a source of uncertainty in the public program.

I use the random utility framework, in which individuals derive utility from alternatives based on observables and unobservables (see McFadden 1981). Preferences are revealed through the observed choice.[7] This framework is useful because it allows me to use the available information about the expected pensions and the choice of the insured without having to assume homogeneity across individuals that have the same expected pension benefit level.

The insured are random utility maximizers who choose to affiliate with one of the two pension programs.[8] I use the rate of return as a measure for future pension wealth, because it eliminates the need to assume discount rates (as needed if present values were used) and of assumptions necessary to compare the different pension payments in the two programs. My approach only makes the assumption that the insured do not have a systematic preference for the so-called scheduled withdrawal form of pension payment in the private program, in which the amount of the pension decreases with increasing age. If other measures of pension wealth were used - such as pension wealth at the age of retirement - additional assumptions such as the cost of the annuity in the public program as well as the discount rates would become necessary.

The rate of return in the public program depends on the IRR, denoted l_{as}, the interest rate that equates the present value of contributions and the present value of future pension benefits. Because of the way in which the pension in the public program is calculated, the IRR depends on age and sex (see Appendix 1 for the calculation in detail). The insured expect to live for the average life expectancy. The IRR is subject to a default risk, d_{as}, that also depends on age and sex. The default risk results from the financial imbalances of the public program, which induce uncertainty about the future ability of the

[7] See Wolf and Pohlman (1983) for a study of recovered preferences from asset demand.

[8] Note that maximizing the rate of return gives the same result as maximizing future retirement wealth (w): Since $w=(1+r)^y$ * present value w, U(w) is a positive monotonic transformation of U(r), to which the utility ordering is invariant (see Jehle and Reny 2001: 17).

public program to pay a part or all of the promised pensions. The rate of return in the public program, r_{as}^{pub}, is then defined as

$$r_{as}^{pub} = l_{as}(1 - d_{as}),\ d_{as} e [0,1]\ . \qquad (1)$$

The rate of return on the accumulated funds in the private program is the market return achieved by the pension fund administrator, r_m. The return in the private program depends also on the return on the recognition bond, which is denoted ?. I assume that the insured have the same expectation over the default risk on this return as on the internal rate of return of the public program. For the insured who will receive both r_m and g, the rate of return in the public program is composed of the weighted shares of the returns on the recognition bonds and the accumulated funds. For analytical tractability it is assumed that the insured take into account only the relative number of years they contributed into a program. Let this weight, b_{as}, be defined as

$$b_{as} = \frac{z}{retirement\ age - 19}\ , \qquad (2)$$

$$where\ z = \begin{cases} age - 19\ if\ age > 22 \\ 0\ otherwise \end{cases}.$$

This assumes an initial working age of 19 years and takes into account the fact that the insured are only eligible for recognition bonds if they have contributed during at least 150 weeks into the public program.

The rate of return in the private program, r_{as}^{prv}, is then specified by

$$r_{as}^{prv} = g b_{as}(1 - d_{as}) + r_m(1 - b_{as}), where\ b_{as} e\ [0,1]. \qquad (3)$$

The utility function is, in accordance with the random utility model, additively separable and decomposable into two parts: a systematic part, $U(r_{as})$, and a stochastic component, e_i, which represents a random, unobservable effect associated with the utility of the alternative for the insured. Let

$$V\left(r_i^x\right)=U\left(r_{as}^x\right)+e_i \tag{4}$$

be the total utility for individual i in program $x\in\{prv, pub\}$.

The systematic part of the utility function is assumed to be quadratic:

$$U\left(r_{as}^x\right)=ar_{as}^x+b\left(r_{as}^x\right)^2+c,$$

where $a>0,\ b<0,\ c>0$ and $r\le-\frac{a}{2b}$, such that the marginal utility from interest is always positive. Assuming a quadratic utility function is useful because it avoids the need for assumptions about the distribution of the outcomes, since the expected utility depends solely on the mean and variance of the rate of return:

$$EU\left(r_{as}\right)=a\bar{r}_{as}+b\bar{r}_{as}^{\,2}+bs_{r_{as}}^2+c. \tag{5}$$

Then

$$EU\left(r_{as}^{pub}\right)=al_{as}\left(1-\bar{d}_{as}\right)+bl_{as}^2\left(\left(1-\bar{d}_{as}\right)^2+s_{d_{as}}^2\right) \tag{6}$$

and

$$\begin{aligned}EU\left(r_{as}^{prv}\right)=&a\left[gb_{as}\left(1-\bar{d}_{as}\right)+\left(1-b_{as}\right)\bar{r}_m\right]+\\&b\left[g^2b_{as}^2\left(\left(1-\bar{d}_{as}\right)^2+s_{d_{as}}^2\right)+2gb_{as}\left(1-b_{as}\right)\bar{r}_m\left(1-\bar{d}_{as}\right)+\left(1-b_{as}\right)^2\left(s_{r_m}^2+\bar{r}_m^2\right)\right]+c.\end{aligned} \tag{7}$$

The insured choose the public program if its utility is higher than in the private program, i. e. if $V\left(r_i^{pub}\right)\ge V\left(r_i^{prv}\right)$.

Taking expectations yields:

$$EU\left(r_{as}^{pub}\right)+e_i\ge EU\left(r_{as}^{prv}\right),$$

where $e_i=e_i^{pub}-e_i^{prv}$.

Define a latent variable, y_i^*, as the difference in expected utilities such that

$$y_i^*=EU\left(r_{as}^{pub}\right)-EU\left(r_{as}^{prv}\right)+e_i. \tag{8}$$

Let $y_i = \begin{cases} 1 \; if \;\; y_i^* \geq 0 \quad (insured \; chooses \; the \; public \; program) \\ 0 \; otherwise \; (insured \; chooses \; the \; private \; program) \end{cases}$.

The selection criteria is then

$$\Pr(y_i = 1) = \Pr\left(EU\left(r_{as}^{pub}\right) \geq EU\left(r_{as}^{prv}\right) - e_i\right) = \Pr\left(y_i^* \geq 0\right). \tag{9}$$

And

$$y_i^* = a\left[l_{as}\left(1-\bar{d}_{as}\right) - gb_{as}\left(1-\bar{d}_{as}\right) - \left(1-b_{as}\right)\bar{r}_m\right] + \\ + b\left[\left(l_{as}^2 - g^2 b_{as}^2\right)\left(\left(1-\bar{d}_{as}\right)^2 + s_{\bar{d}_{as}}^2\right) - 2gb_{as}\left(1-b_{as}\right)\bar{r}_m\left(1-\bar{d}_{as}\right) - \left(1-b_{as}\right)^2\left(s_{\bar{r}_m}^2 + \bar{r}_m^2\right)\right] + e_i \tag{10}$$

or

$$y_i^* = aX_{as} + bZ_{as} + e_i \tag{11}$$

4 Specification

4.1 Econometric Specification

I will use a binary probit model with heteroskedastic error terms. According to the model, the insured choose the pension program whose return gives the higher expected utility.

There might be systematic differences in the choice among different groups of insured. The insured with a higher level of education might have a better understanding of the way the private program works or, more interesting, a better understanding of the fact that because switching is allowed, the internal rate of return of the public program acts as a de-facto, even though somewhat risky, lower bound. Since older insured will benefit less from the switching possibility, age at a given educational level should also have a systematic impact on the choice. There is also evidence that better educated insured in Colombia make a more financially beneficial pension program choice (Kleinjans 2003b). For example, higher educated women (who benefit relatively more from the public program than men because of their higher life expectancy and lower

retirement age) and higher educated insured with a low salary (who expect to benefit from the minimum pension) are more likely to choose the public program then their less educated counterparts. This would explain the fact that affiliation with the public program decreases with higher educational achievement (see Table 3). At the same time, the decision might be influenced by the salary level of the insured. A higher salary might indicate that the insured have more experience with and confidence in financial markets, and thus are generally more likely to choose the private program (see Table 3). These systematic differences result in heteroskedasticity of the error term.

Let v_i be the vector of the characteristics believed to influence choice: $v_i = (educa\ eduage\ sal0000)$,

where *educa* is the educational level, *eduage* education * age, and *sal0000* the salary of the insured in 10,000 pesos.

I specify the variance of the error term, following Harvey (1976), as:

$$s_i^2 = [\exp(l'v_i)]^2,$$

where l is the vector of the coefficients to be estimated. If $\hat{l} > 0$, the estimated relationship between the variance and the characteristic is positive. Note that if $l = 0$ then $s_i^2 = 1$. I expect the variance to be bigger for better educated and insured with higher salaries and smaller for older insured at any given educational level.

The log-likelihood function is then

$$\ln L = \sum_{i=1}^{n} \left\{ y_i \ln \Phi\left(\frac{aX_{as} + bZ_{as}}{\exp(l'v_i)} \right) + (1 - y_i) \ln \left[1 - \Phi\left(\frac{aX_{as} + bZ_{as}}{\exp(l'v_i)} \right) \right] \right\},$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. I can then estimate the utility parameters $\hat{a}, \hat{b}$ and the vector of the variance coefficients $\hat{l}$, where I expect to find $\hat{a} > 0, \hat{b} < 0,\ and\ r_{as} \leq -\frac{\hat{a}}{2\hat{b}}$.

4.2 Assumptions on the Parameters

To be able to estimate $\hat{a}$ and $\hat{b}$, I have to calculate and make assumptions on l_{as}, b_{as}, $\bar{d}_{as}$ *and* $s^{2}_{d_{as}}, \bar{r}_m$ *and* $s^{2}_{r_m}$. The real annual interest on the recognition bonds is ? = 4%, as determined by law.

The internal rate of return (IRR), l_{as}, is the interest rate that equates the present value of contributions and benefits (see Appendix 1 for details of the calculation). It depends on the number of weeks during which contributions were made (which in turn depends on the initial working age, the retirement age, and the density of contributions), the life expectancy at retirement, and wage growth. The contribution density is the percentage of the work life during which the insured contributes and is assumed to be constant. It is smaller than one due to time spend out of the formal labor force for reasons such as child rearing, unemployment or work in the informal sector. The minimum contribution time to be eligible for a pension is 1000 weeks. If then the pension is below the minimum wage level, a minimum pension guarantee equal to the minimum wage takes effect. The retirement age used is 57 years for women and 62 years for men, which assumes that the insured expect a planned retirement age increase will take place. For simplicity, I make the additional assumption that the initial working age is 19 years. For the resulting IRR's depending on the density of contributions, see Table 1 in Appendix 2.

The issue of the minimum pension guarantee is complicated. In addition to the question of who will benefit from it in the public program, there is also a minimum pension in the private program; however, its eligibility requirement is stricter (at least 1150 weeks contributed) and the pension is lower since it is paid only 12 times a year. Therefore, I will assume that the insured who expect to receive a minimum pension compare the IRR in the public program of a minimum pension with the same return in the private program as previously assumed, r_{as}^{prv}. I also assume that, regardless of the wage growth of the insured not expecting to benefit from the minimum pension guarantee, there is no wage growth for those insured, since it is sensible to expect in general less or no wage growth for the lowest wage bracket of unskilled workers.

The rates of return in the public program are very high. Because of this, the insured expect the state to default on its promises with positive probability. This expected default risk will be the higher the younger the insured are, since the financial imbalances of the public program will increase over the years. In addition, the past reform of 1994 excluded the insured above age 34 (women) and 39 (men), which decreases the expected default risk for older insured. I model the expected risk of default, therefore, as decreasing with age:

$$\bar{d}_{am} = 0.4\frac{15^2}{age^2}$$

$$\bar{d}_{af} = 0.45\frac{15^2}{age^2}$$

and $s^2_{d_{as}} = \left(\frac{\bar{d}_{as}}{2}\right)^2$

$\bar{d}_{am}$ will then range between 0.4 and 0.051, and $\bar{d}_{af}$ between 0.45 and 0.074. This means that the insured expect the IRR on average to be only between 60% and 95% (men) and 55% and 93% (women) of the promised IRR. This specification implies that younger insured expect strong cuts in benefits (or increases in the contribution rates) to occur, while older insured expect only small cuts.[9] The higher expected default risk of women, $\bar{d}_{af}$, reflects the likelihood that the five year lower retirement age for women may be increased relative to men's.

The constant annual wage growth is 2%. Both men and women achieve under these assumptions a replacement rate of 85%, and all of the insured of which 85% of their salary is below the minimum wage will expect to receive the minimum pension, which

[9] This does not imply that, in expectation, the program will be in steady state after a default or a reform. This is not necessary or desirable since individuals will continue to expect a strong involvement of the state in the pension system (which could include subsidies) as well as a continued increase in coverage.

are the insured that earn less than 1.176 times the minimum wage. Life expectancy at retirement is 22 years for men and 29 years for women.[10]

5 Estimation

Given the assumptions on the parameters, I can now estimate the choice of the insured. In the basic model, I consider $\bar{r}_m = 4\%, s^2_{r_m} = 30$ and the contribution density 0.8. The variance is modeled in accordance to the variance of the returns achieved by the oldest private pension program in Latin America, in Chile. There, between 1981 and 2001, the average returns achieved were 10.7% with a variance of 74.76. Since these high interest rates are most probably the result of the economic circumstances at that time and the economic development of Chile, I assume lower interest rates, and an accordingly lower variance of the returns. The assumed contribution density takes into account periods of, for instance, unemployment during which the insured do not contribute to the system. The results of the estimation are shown in Table 2 under Model II; Model I shows the estimation without the correction for heteroskedasticity.

Compared to the estimated model without heteroskedasticity (see Table 2 Model I), the log-likelihood increases as well as the statistical significance of $\hat{b}$. Note that both $\hat{a}$ and $\hat{b}$ have very different values. Without controlling for heteroskedasticity, the model over-predicts affiliation with the public program for the insured with the highest educational level, in the lowest age group, and with a high salary, and under-predicts affiliation with the public program for the insured with a low or medium educational level, in the oldest age group, and with low salaries.

---

[10] The life expectancy at age 60 in 1995-2000 is 21 years for women and 18 years for men (UN 1999). Consecuently, the life expectancies at retirement are about 24 years (women) and 16 years (men) at the considered retirement ages of 57 (women) and 62 (men) in 1995-2000. The here assumed life expectancies of 29 years (women) and 22 years (men) assume that the trend of the increase in life expectancy continues and correspond to a 30 year old in 1997 (see Kleinjans 2002 for more details).

Table 2: Heteroskedastic Probit Model of Pension Program Choice (coefficients)

Explanatory Variable	**Model I**	**Model II**	**Model III**	**Model IV**
	Basic Model without heteroskedasticity	**Model I with heteroske-dasticity**	**Model II with** $r_m = 5\%$	**Model II with** $s^2_{r_m} = 36$ **and** $r_m = 6\%$
$\hat{a}$ **(a842nmp)**	0.145 (0.019)*	1.075 (0.337)*	1.396 (0.450)*	1.557 (0.550)*
$\hat{b}$ **(b8492nmp)**	-0.0056 (0.0029)	-0.061 (0.026)**	-0.085 (0.029)*	-0.095 (0.030)*
$\hat{l}$ **(educa)**		1.750 (0.434)*	1.182 (0.430)*	1.944 (0.429)*
$\hat{l}$ **(*eduage*) (educa*age)**		-0.049 (0.012)*	-0.051 (0.012)*	-0.054 (0.012)*
$\hat{l}$ **(sal0000) (salary in 10000 pesos)**		0.044 (0.010)*	0.044 (0.010)*	0.044 (0.010)*
Log Likelihood	-960.67	-931.01	-931.95	-931.93
Prob>chi		0.0061	0.0062	0.0063

Standard errors in parenthesis. * and ** denote a level of statistical significance of 1% and 5%, respectively.

Changing the variance of the market return worsens the fit of the model. Variations of the contribution density (lower density, lower density for women or the insured expecting to benefit from the minimum pension) and lowering the expected mean market return also worsens the fit of the model. A market return higher than 4% changes the fit only slightly, but changes the coefficients to a high degree. Model III and Model IV (see Table 2) show the best fitting models with these higher returns.

Table 3 Predicted Probabilities Versus Observed Frequencies[a]

		Total	**Educa =1**	**Educa =2**	**Educa =3**	**Female**	**Male**	**age 15-25**	**age 26-35**	**age 36-42**	**salary < 202359**	**202359 <salary < 344011**	**salary > 344010**
Number of Obser-vations		1430	210	827	393	530	900	331	707	392	416	463	551
	Observed Frequency	.6119	.7190	.6203	.5369	.6226	.6056	.5831	.5941	.6684	.6995	.6112	.5463
Model II	**Predicted Probability**	.6148 (.0819)	.7006 (.0690)	.6217 (.0701)	.5545 (.0622)	.6254 (.0914)	.6086 (.0751)	.5792 (.0486)	.6096 (.0741)	.6542 (.0998)	.6790 (.0773)	.6363 (.0613)	.5483 (.0427)
Model III	**Predicted Probability**	.6147 (.0820)	.7010 (.0687)	.6216 (.0700)	.5541 (.0624)	.6256 (.0920)	.6083 (.0748)	.5799 (.0498)	.6093 (.0742)	.6538 (.0995)	.6782 (.0775)	.6369 (.0617)	.5481 (.0428)
Model IV	**Predicted Probability**	.6146 (.0821)	.7014 (.0681)	.6213 (.0698)	.5539 (.0633)	.6259 (.0927)	.6079 (.0744)	.5808 (.0518)	.6089 (.0745)	.6533 (.0991)	.6772 (.0772)	.6375 (.0628)	.5481 (.0432)

[a] Standard deviations in parenthesis.

All models that control for heteroskedasticity fulfill the condition for positive marginal utility of $r \leq -\frac{a}{2b}$. The predictive power of all three models with heteroskedasticity correction is very good, both for the entire sample and for subsamples, supporting the thesis that the choice of the insured is heavily influenced by the expected returns. Table 3 shows a comparison of the observed frequencies and the predicted probabilities for the entire sample and subsamples by sex, educational level, age groups, and salary groups. All of the predicted probabilities are well below one standard deviation from the observed frequencies.

6 Prediction of Outcomes from a New Pension Reform Proposal

To get insight in how the insured would adjust their choice if a pension reform took place, I consider the effects of an existing pension reform proposal on the expected future pension income and predict the new choices using the in the previous section estimated parameters.

6.1 The Reform Proposal

The newly elected government of Andrés Pastrana brought a proposal before congress in its first month in office that would reform the pension system. The proposal includes, as expected, benefit cuts and increases in retirement ages in the public program. The most important changes regarding the choice of the insured between the private and the public program would be as follows:

- An increase in the retirement age: from 55/60 until 2008, to 58/62 in 2009, and 62/65 in 2018;
- A decrease in the replacement rate from 65%-85% to 60%-80% in 2009;
- An increase in the contribution period from between 1000 - 1400 weeks to 1200-1800 weeks in 2009 and 1300-1800 in 2018;

- An increase of the contribution rate of 13.5% to 14.5% in 2004 and 15.5% in 2005. In the private program, the additional 2% of the payroll will be used to finance the minimum pension, which the state stops to guarantee.

6.2 The New Decision Problem

This proposed reform would affect the expected future pension income. The increase in the contribution rate, however, would not change the decision of the insured, since the additional contribution in the private program does not enter the individually accumulated funds.

The to 58 (women) and 62 years (men) increased retirement age in 2009 would not affect any of the insured in the sample, since none of them will retire before 2014. Hence the new retirement age all of them I will consider in their decision is 62 (women) and 65 (men).

The relevant replacement rates would lie between 60% and 80%, and the corresponding contribution periods between 1300 and 1800 weeks. Under the same assumption about initial working age and density of contribution as before, all of the insured will receive a replacement rate of 80%.

Therefore, the internal rates of return in the public program would decrease (because of the lower replacement rate, the higher contribution periods, and the higher retirement age). Also, the risk of default in the public program should now be lower. In addition to the initial default expectation, I will use three different new specifications of the default risk, d_{as}, since the predicted changes in probabilities should be very sensitive to the changes on expectation about future defaults:

(d1) $\bar{d}_{as} = 0$

(d2) $\bar{d}_{am} = 0.2\frac{15^2}{age^2}$ and $\bar{d}_{af} = 0.22\frac{15^2}{age^2}$ and $s^{\,2}_{d_{as}} = \left(\frac{\bar{d}_{as}}{2}\right)^2$

(d3) $\bar{d}_{am} = 0.3\frac{15^2}{age^2}$ and $\bar{d}_{af} = 0.32\frac{15^2}{age^2}$ and $s^2_{\bar{d}_{as}} = \left(\frac{\bar{d}_{as}}{2}\right)^2$.

The default risk for women decreases relatively more than for men since the reform increased the retirement age relatively more for women than for men. I assume that women at age 62 have a life expectancy of 24 years and men at age 65 of 19 years, decreasing the initially assumed life expectancy by the same number of years the retirement age increased.

The resulting new internal rates of return are, for a density of 0.8, 6.16% for women without the benefit from the minimum pension guarantee and 5.51% with the benefit; and 5.41% and 4.70% for men, respectively. Note that the internal rates of return decreased, but only by maximally 1.2%-points, so that they are still well above the steady-state.

Since the replacement rates changed, more of the insured will expect to receive a minimum pension. In the following, I assume that the insured that earn less than 1.25 times the minimum wage expect to benefit from the minimum pension guarantee, which are the insured with wages such that the 80% replacement rate would yield a pension below the minimum pension. In the following it is assumed that the change in the pension legislation will not change the expectations about the future market interest or its variance.

6.3 Results

The changes in the public pension program have two effects: First, the private program becomes relatively more beneficial, so that some insured should be inclined to switch to it from the public program; second, the expected default risk in the public program will increase and incline its affiliates to stay.

I find that the first effect is dominating because fewer of the insured than before are predicted to choose the public program. The change in probabilities, however, is very small. This is surprising since the policy simulation assumes that the benefits in the

public program are cut and the retirement ages increased considerably; the internal rates of return decreased by about 13% for men and 17% for women. Assuming that the expected default risk remains unchanged, between 1.9% and 5.5% more of the insured are expected to choose the private program, depending on the model. These percentages, though, decrease significantly if the expected default risk decreases, to less than 2% in the case of all models and all considered specifications of the new default risk. A higher market interest lowers the percentage of the insured that choose the public program, but only by a small percentage. In the extreme case of no expected default risk, less than 1% of the insured are expected to switch (see Table 4).

Table 4: Changes in Predicted Pension Program Choice Probabilities Resulting from Proposed Changes in the Pension Legislation [a]

	Baseline Probability	**New Probability with (d1)**	**New Probability with (d2)**	**New Probability with (d3)**	**New Probability with unchanged d**
Model II	.6148 (.082)	.6077 (.074) (-.0071)	.6020 (.071) (-.0128)	.5990 (.070) (-.0158)	.5954 (.0680) (-.0194)
Model III	.6147 (.082)	.6076 (.073) (-.0071)	.6010 (.070) (-.0137)	.5974 (.068) (-.0173)	.5696 (.0504) (-.0451
Model IV	.6146 (.082)	.6076 (.074) (-.0070)	.6001 (.069) (-.0145)	.5961 (.067) (-.0185)	.5598 (.0421) (-.0548)

Frequency of the public program chosen: .6119.

[a] Standard deviation and change in probability in parenthesis.

Looking at the expected probabilities of the subsamples, I find a consistent decrease in the probabilities to affiliate with the public program, with patterns similar to the whole sample. Increasing the expected default risk decreases affiliation with the public program as does increasing the expected market interest. The only exception is the youngest group of insured, where a higher market interest increases the expected

probability of affiliating with the public program (see Table 6). Note, though, that the standard deviations are high. The insured with the highest level of education react even less strongly in absolute terms to the change in the pension legislation than the insured with less education (see Table 5). Likewise, younger insured react less strongly (see Table 6), as do the insured with high salaries (see Table 7).

It is not clear why the predicted response is so small, and there is no distinguishable pattern of certain types of the insured to be more prone to choose the private program after the proposed reform than others. When I reduced the internal rates of return by more than the proposed reform would, by one third (assuming that the same insured expect to receive the minimum pension than with the reform proposed), the predicted probability of choosing the public program decreased more significantly, by another 4% in the case of d2 and Model 2. In this case, I found that a clearer pattern emerged. The probability of the insured with low wages, low education, and higher age to choose the public program decreased by over 10%, much more than that of other groups. This means that the groups that have a higher risk aversion are choosing the public program less than before.

These results suggest that changes in the public pension program will not affect the decision of the insured in a significant way. The effect can be further reduced if the reforms conducted increase the confidence in the public program and thus decrease the expected default risk. This means that the Colombian government could most likely improve the financial imbalance of the public program by cutting benefits (within certain limits) with little danger that the insured – especially those with higher salaries – would flock to the private program, decreasing the contributions to the public program and consequently worsen its financial instabilities. It would be important for the government to at the same time make it clear to the public that the reform will sustainably improve the financial situation of the public program, and therefore decrease the expected default risk.

Table 5: Predicted Pension Program Choice Probabilities Resulting from Proposed Changes in the Pension Legislation by Educational Level and Expected Default Risk (standard deviations in parenthesis)

Model	Educa==1			Educa=2			Educa=3		
	Baseline Probability	**New Probability with (2)**	**New Probability with (3)**	**Baseline Probability**	**New Probability with (2)**	**New Probability with (3)**	**Baseline Probability**	**New Probability with (2)**	**New Probability with (3)**
II	.7006 (.069)	.6774 (.059)	.6720 (.059)	.6217 (.070)	.6086 (.060)	.6052 (.059)	.5545 (.062)	.5480 (.054)	.5469 (.053)
III	.7010 (.069)	.6750 (.059)	.6685 (.058)	.6216 (.070)	.6076 (.059)	.6037 (.058)	.5541 (.062)	.5475 (.054)	.5462 (.053)
IV	.7014 (.068)	.6737 (.059)	.6662 (.058)	.6213 (.070)	.6067 (.058)	.6024 (.056)	.5539 (.063)	.5469 (.054)	.5454 (.052)

Table 6: Predicted Pension Program Choice Probabilities Resulting from Proposed Changes in the Pension Legislation by Age Group and Expected Default Risk (standard deviations in parenthesis)

Model	Age 15-25			Age 26-35			Age 36-42		
	Baseline Probability	**New Probability with (2)**	**New Probability with (3)**	**Baseline Probability**	**New Probability with (2)**	**New Probability with (3)**	**Baseline Probability**	**New Probability with (2)**	**New Probability with (3)**
II	.5792 (.049)	.5756 (.046)	.5711 (.043)	.6096 (.074)	.5977 (.066)	.5948 (.064)	.6542 (.0998)	.6322 (.086)	.6300 (.084)
III	.5799 (.050)	.5762 (.047)	.5712 (.044)	.6093 (.074)	.5968 (.065)	.5934 (.063)	.6538 (.0995)	.6294 (.084)	.6267 (.082)
IV	.5808 (.052)	.5773 (.049)	.5721 (.046)	.6089 (.075)	.5963 (.065)	.5924 (.063)	.6533 (.0991)	.6264 (.082)	.6229 (.080)

Table 7: Predicted Pension Program Choice Probabilities Resulting from Proposed Changes in the Pension Legislation by Salary Level and Expected Default Risk (standard deviations in parenthesis)

Model	Salary < 202359			202358< Salary < 344011			Salary > 344011		
	Baseline Probability	New Probability with (2)	New Probability with (3)	Baseline Probability	New Probability with (2)	New Probability with (3)	Baseline Probability	New Probability with (2)	New Probability with (3)
II	.6790 (.077)	.6598 (.065)	.6543 (.065)	.6363 (.061)	.6210 (.051)	.6177 (.051)	.5483 (.043)	.5425 (.037)	.5416 (.036)
III	.6782 (.078)	.6568 (.064)	.6503 (.064)	.6369 (.062)	.6207 (.050)	.6169 (.050)	.5481 (.043)	.5422 (.037)	.5411 (.036)
IV	.6772 (.077)	.6547 (.064)	.6474 (.063)	.6375 (.063)	.6204 (.050)	.6161 (.050)	.5481 (.043)	.5418 (.037)	.5405 (.036)

7 Conclusions

This paper develops a structural model of choice between two types of pension programs, a public and a private one, to shed light on how much of the choice can be explained as the outcome of a maximization of future expected pension income. It takes into account two kinds of risk: the risk that the state defaults on its pension promises in the public program (by, for example, reforming it for those currently insured); and the capital market risk in the private program. The insured receive utility from the interest rate of the pension program and choose the one that renders the higher expected utility. I find that an empirical estimation of this choice yields highly statistical significant coefficients, supporting the importance of the maximization of future expected pension income for the choice. I find heteroskedasticity of the error term for three independent variables, education, education*age, and salary, for which I correct by estimating variance coefficients for each of these variables.

Using the estimated coefficients from the model, I simulate the behavior of the insured in the case that a proposed pension reform of the Colombian government would be implemented. I find that very few of the insured are expected to switch from the public to the private program as a result of the pension reform, and that the changed expectations about the default risk in the public program play an important role in determining the number of the insured that will switch to the private program. This means that the Colombian government could improve the financial situation of the public program significantly by cutting benefits and increasing retirement ages, without endangering these gains by causing a high number of the insured to conduct the costly switch to the private program.

References

Aaron, H. (1966). The Social Insurance Paradox. *Canadian Journal of Economics and Political Science* 32 (3), 371-374.

Augusztinovics, M. et al (2002). The Hungarian Pension System Before and After the Reform of 1998. In: Fultz, E. (ed.): *Pension Reform in Central and Eastern Europe. Restructuring with Privatization: Case Studies of Hungary and Poland.* Budapest: ILO, 25-93.

Benartzi, S. and R. H. Thaler (2001). Naive Diversification Strategies in Defined-Contribution Saving Plans. *American Economic Review* 91 (1), 79-98.

Brugiavini. A. and R. Disney (1995). The Choice of Private Pension Plan Under Uncertainty. *IFS and ESRC Working Paper Series* No. W95/5.

Choi, J.J., D. Laibson, B C. Madrian, and A. Metrick (2001a). Defined Contribution Pensions: Plan Rules, Participant Decisions, and the Path of the Least Resistance. *National Bureau of Economic Research Working Pape*r 8655.

Choi, J. J., D. Laibson, B.C. Madrian, and A. Metrick, 2001b. For Better or For Worse: Default Effects and 401(k) Savings Behavior. *National Bureau of Economic Research Working Paper* 8651.

DANE (2002). Information available on the website www.dane.gov.co. Santafé de Bogotá.

Disney, R., R. Palacios and E. Whitehouse (1999). Individual Choice of Pension Arrangement as a Pension Reform Strategy. *IFS Working Paper Series* No. W99/18.

Economic Commission for Latin America and the Caribbean (ECLAC) (1998). *Statistical Yearbook for Latin America and the Caribbean: 1997*. LC/G.1987-P. Santiago de Chile (February).

Feldstein, M. (1997). Insulations of Pensions from Political Risk. In: Valdés-Prieto, S. (ed.): *The Economics of Pensions: Principles, Policies, and International Experience*. Cambridge, NY: Cambridge University Press, 33-57.

Geanakoplos, J., O. S. Mitchell, and S. P. Zeldes (1999). Social Security Money's Worth. In: Mitchell, O.S., R. J. Myers, and H. Young (eds.): *Prospects for Social Security Reform.* Philadelphia: University of Pennsylvania Press, 79-151.

Gustman, A. L. and T. L. Steinmeier (1998). Privatizing Social Security: First-Round Effects of a Generic, Voluntary, Privatized Social Security System. In: Feldstein. M. (ed.): *Privatizing Social Security*. Chicago and London: The University of Chicago Press, 313-357.

Harvey, A. C. (1976). Estimating Regression Models with Multiplicative Heteroskedasticity, *Econometrica* 44 (3), 461-465.

Iyer, S. (1999). *Actuarial mathematics of social security pensions*. Geneva: International Labour Office.

Jehle, G. and P. Reny (2001). *Advanced Microeconomic Theory*. Boston: Addison Wesley.

Kleinjans, K. (2002). *Determinants of Pension Choice: The Case of Colombia*. Manuscript.

Kleinjans, K. (2003a forthcoming). The Colombian Pension System After the Reform of 1994: Evaluation and Perspectives. *International Social Security Review* 56 (1).

Kleinjans, K. (2003b forthcoming). Do individuals make informed pension program choices? Evidence from Colombia. In: E. Overbye and P. Kemp, *Pensions: Challenges and Reform*, Ashgate: Aldershot.

Kotlikoff, L. J., K. A. Smetters, and J. Walliser (1998). Opting out of Social Security and Adverse Selection. *National Bureau of Economic Research Working Paper* 6430.

Leimer, D. R. (1995). A Guide to Social Security Money's Worth Issues. *Social Security Bulletin* 58 (2), 3-20.

Madrian, B., and D. F. Shea (2001). The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior. *The Quarterly Journal of Economics* CXVI (4), 1149-1187.

McFadden, D. (1981). Econometric Models of Probabilistic Choice. In: Manski, C. F. and D. McFadden. *Structural Analysis of Discrete Data with Econometric Applications*. Cambridge: MIT Press.

McHale, J. (1999). The Risk of Social Security Benefit Rule Changes: Some International Evidence. *National Bureau of Economic Research Working Paper* 7031.

Palacios, R., and E. Whitehouse (1998). The Role of Choice in the Transition to a Funded Pension System. *The World Bank Social Protection Discussion Paper* No. 9812, Washington, D.C.: The World Bank.

Samuelson, P. A. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy* 66 (3), 467-482.

Samuelson, W., Zeckhauser, R. (1988). Status Quo Bias in Decision Making. *Journal of Risk and Uncertainty* 1, 7-59.

Samwick, A. (1998). Discount Heterogeneity and Social Security Reform. *Journal of Development Economics* 57, 117-146.

Srinivas, P. S. and J. Yermo (1999). Do Investment Regulations Compromise Pension Fund Performance? Evidence from Latin America. *World Bank Latin American and Caribbean Studies Viewpoints*. Washington, D. C.: The World Bank.

Turner, J. A. and D. M. Rajnes (2001). Rate of Return Guarantees for Mandatory Defined Contribution Plans. *International Social Security Review* 54 (4), 49-66.

Wolf, C. and L. Pohlman (1983). The Recovery of Risk Preferences From Actual Choices. *Econometrica* 51 (3), 843-850.

World Bank (2000). *World Development Report 2000/2001*. Washington, D. C.: Oxford University Press.

Appendix 1: The Calculation of the IRR

The IRR of a defined-benefit program is defined as the interest rate that equates the present value of contributions, PVB(i), and the present value of benefits, PVC(i), received. In the following, the reference time for the insured is age 19. The calculations assume that the insured is affiliated with the public program during her or his entire work life (this appendix follows closely the appendix in Kleinjans 2002).

Benefits

In the Colombian public pension program the pension benefit formula is specified as:

$$P = w(10)r\,, \qquad \text{(A1)}$$

where P is the monthly pension, $w(10)$ is the monthly average wage of the last 10 years before retirement, and r is the replacement rate. The replacement rate is the ratio between the pension and $w(10)$, and is legally determined depending on the number of contributed weeks during working life, which in this calculation is defined as follows:

$$t = (a - 19) \cdot \left(\frac{365.25}{7} \right) \cdot d \,, \tag{A2}$$

where a is the retirement age, 19 the assumed initial working age, $\left(\frac{365.25}{7} \right)$ gives the average number of weeks per year, and d is the density of contributions. The latter is the percentage of the working life during which the insured contributed to the pension program and is assumed to be uniform over the work life. The pension law determines the replacement rate as follows:

$$If\ t < 1000\ \ then\ r = 0 \tag{A3}$$

$$if\ 1000 \le t < 1050\ then\ r = 0.65$$
$$if\ 1050 \le t < 1100\ then\ r = 0.67$$
$$if\ 1100 \le t < 1150\ then\ r = 0.69$$
$$if\ 1150 \le t < 1200\ then\ r = 0.71$$
$$if\ 1200 \le t < 1250\ then\ r = 0.73$$
$$if\ 1250 \le t < 1300\ then\ r = 0.76$$
$$if\ 1300 \le t < 1350\ then\ r = 0.79$$
$$if\ 1350 \le t < 1400\ then\ r = 0.82$$
$$if\ t > 1400\ then\ r = 0.85.$$

This pension as calculated in (A1) is paid in each of the 12 calendar months and, in addition, in June and December in each year, so that the total number of pension payments per year is 14. In the following it is assumed that the twelve monthly pension payments are made at the beginning of each month, and the two additional payments at the end of June and December. An insured is eligible for pension if at reaching the legal pension age she contributed during at least 1000 weeks over her working life. In the

following, the case that the insured does not reach the minimum contribution time is omitted.

The state guarantees a minimum pension in the amount of the legal minimum wage under two conditions: (i) the insured is eligible for pension; and (ii) the pension as calculated in (A2) is lower than the legal minimum wage. The calculations assume that contributions are made uniformly over the working life. The average wage of the last ten years, assuming constant yearly wage growth, can be calculated as follows:

$$w(10) = \left(\frac{w(1)}{10}\right)(1+g)^{(a-19-11)}\left(\frac{1-(1+g)^{10}}{-g}\right), \qquad \text{(A4)}$$

where $w(1)$ is the initial wage, g is the constant yearly wage increase, and

$$\left(\frac{1-(1+g)^{10}}{-g}\right) = 1+(1+g)+(1+g)^2+\ldots+(1+g)^9 .$$

The present value of the benefits (PVB) if the minimum pension guarantee does not hold can be calculated using standard actuarial techniques (see Iyer 1999) as follows:

$$PVB = rw(10)\left[1-\left(\frac{1}{1+i}\right)^{y}\right]\left(\frac{1}{1+i}\right)^{(a-19)}\left[\frac{(1+i)^{\frac{1}{12}}}{12\left[(1+i)^{\frac{1}{12}}-1\right]}+\frac{1}{2\left[(1+i)^{\frac{1}{2}}-1\right]}\right] \qquad \text{(A5)}$$

where i is the annual discount rate, which becomes the IRR once PVB and PVC are set equal. Compounding periods are monthly, and y is the life expectancy in years at retirement. In the case the minimum wage guarantee holds, $rw(10)$ is replaced by the legal monthly minimum wage. In the case with zero wage growth, $w(10)$ is replaced by $w(1)$.

Contributions

The contribution rate (disregarding administration cost and contributions to the disability and survivor insurance) is 10% of the monthly salary. Most formally employed Colombians receive 14 monthly salaries per year, one every calendar month and an

additional salary in June and December. In the following, it is assumed that the insured made 14 payments per year, twelve at the beginning of each month and one each at the end of June and December. The assumed initial working age is 19 years. Then the present value of contributions (PVC) can be calculated using standard actuarial techniques (see Iyer 1999) as follows:

$$PVC = 0.1 \cdot d \cdot \frac{i}{1+i}\left[\frac{(1+i)^{\frac{1}{12}}}{12\left[(1+i)^{\frac{1}{12}}-1\right]} + \frac{1}{2\left[(1+i)^{\frac{1}{2}}-1\right]}\right] w(1) \frac{1-(1+g)^{(a-19-1)}\left(\frac{1}{1+i}\right)^{(a-19-1)}}{1-(1+g)\left(\frac{1}{1+i}\right)} \quad \text{(A6).}$$

By setting $PVB = PVC, i = i(d, g)$ can be calculated.

Appendix 2:

Table 1 IRR Under the Current Regulation with and without Minimum Pension Guarantee (MPG) [a]

	Men						**Women**					
Retirement Age (a)	**62 years**				**62 years w/ MPG**		**57 years**				**57 years w/ MPG**	
Life Expectancy at Retirement (q)	**18 years**		**22 years**		**18 years**	**22 years**	**25 years**		**29 years**		**25 years**	**29 years**
Wage Increase (g)	0%	2%	0%	2%	0%	0%	0%	2%	0%	2%	0%	0%
Contribution Density (d)	**Internal Rate of Return**											
0.60	5.21	6.55	5.54	6.84	5.89	6.19	6.32	7.54	6.49	7.69	7.36	7.50
0.70	4.98	6.30	5.31	6.60	5.45	5.76	6.29	7.51	6.46	7.66	6.89	7.05
0.80	4.59	5.90	4.94	6.21	5.06	5.39	6.00	7.20	6.18	7.36	6.49	6.66
0.90	4.25	5.54	4.61	5.87	4.72	5.06	5.64	6.83	5.83	7.00	6.13	6.31
1.00	3.93	5.22	4.31	5.56	4.41	4.77	5.32	6.50	5.52	6.67	5.81	6.00

[a] Initial working age=19 years, contribution $\geq$ 1000 weeks.

Source: Kleinjans (2002).