

University of Regina
 Department of Mathematics and Statistics
 Math 101 - 001 (Fall 2013)
 Final Exam December 10, 2013

Name: _____

Student Number: _____

Instructor: Dr. S. McCann

Instructions: The exam is comprised of two sections:

- Short Answer: Show all your work to receive full marks.
- Choice Section (Choose 4 of 6): Indicate your choice by encircling **keep**.

READ THE EXAM CAREFULLY!! Calculators are not allowed. You will have **180** minutes to complete this exam (max. of **100** points). You may use the back of any page or the last page for extra space.

FOR GRADING PURPOSES (DO NOT WRITE IN TABLE):

Short Answer Questions 1-5: (Chapter 1 Material)	/13
Short Answer Questions 6-8: (Chapter 2 Material)	/14
Short Answer Questions 9-12: (Chapter 3 Material)	/15
Short Answer Questions 13-14: (Chapter 4 Material)	/12
Short Answer Questions 15-18: (Chapter 5 Material)	/14
Choice Section:	/32
Final Exam Grade:	/100

Good Luck!

Short Answer Section (68 points): Show all of your work to receive full marks.

1. (2 points) If an arithmetic progression has second entry 6 and fifth entry 162, then what is the first entry?

2. (2 points) If a geometric progression has second entry 6 and fifth entry 162, then what is the first entry?

3. (2 points) Determine the last digit of the number 3^{10934} ?

$3^1 = 3$	$3^6 = 729$
$3^2 = 9$	$3^7 = 2187$
$3^3 = 27$	$3^8 = 6561$
$3^4 = 81$	$3^9 = 19683$
$3^5 = 243$	$3^{10} = 59049$

4. (2 points) Calculate the sum of the numbers $138 + 139 + 140 + \dots + 862$.

5. (5 points) In a barn, there are pigs and chickens. Each pig has 4 legs and one head. Each chicken has 2 legs and one head. If there are 206 legs and 94 heads, how many chickens and how many pigs are there?

Short Answer Section Continued (68 points):

6. (2 points) Convert the number 543_6 into base-10.
7. (3 points) Convert the number 814_{10} into Mayan numerals. Recall that the Mayan numeral system is a base-20 system where dots = 1, sticks = 5 and an acorn = 0.
8. (9 points) Binary is a base-2 number system where the digits are 0 and 1. Hexadecimal is a base-16 number system where the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $A = 10_{10}$, $B = 11_{10}$, $C = 12_{10}$, $D = 13_{10}$, $E = 14_{10}$, and $F = 15_{10}$. Calculate the following arithmetic operations in binary (base-2) and hexadecimal (base-16) without converting to decimal. Remember to show all your work.

$$(a) \begin{array}{r} 10110110_2 \\ + 1010101_2 \\ \hline \end{array}$$

$$(b) \begin{array}{r} 1\ 2\ 3\ 4_{16} \\ - ABC_{16} \\ \hline \end{array}$$

$$(c) \begin{array}{r} 3\ 1_{16} \\ \times 5B_{16} \\ \hline \end{array}$$

Short Answer Section Continued (68 points):

9. (4 points) Finish the mod 8 multiplication table started below:

\times	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2						6
3	0	3						5
4	0	4						4
5	0	5						3
6	0	6						2
7	0	7	6	5	4	3	2	1

10. (4 points) Find all x satisfying the following equations.

(a) $2x + 6 = 4 \pmod{8}$

(b) $2x^2 = 0 \pmod{8}$

Short Answer Section Continued (68 points):

11. (4 points) Solve the following modular equations.

(a) $5x^2 = 7 \pmod{9}$

(b) $6x = 22x^2 + 11x + 1 \pmod{11}$

12. (3 points) December 10, 2013 is a Tuesday. Recall that leap years (ones with February 29) occur on years that are divisible by 4, that are not divisible by 100 unless also divisible by 400. What day of the week will December 10, 2213 be?

Short Answer Section Continued (68 points):

13. (a) (2 points) Find the prime factorization of 6468.

(b) (2 points) Note that $3024 = 2^4 \times 3^3 \times 7$. How many divisors does 3024 have (i.e., what is $d(3024)$)?

(c) (2 points) Note that $5880 = 2^3 \times 3 \times 5 \times 7^2$. What is $\gcd(3024, 5880)$?

(d) (2 points) What is $\text{lcm}(3024, 5880)$? You do not need to simplify the prime factorization.

14. (4 points) Use the Euclidean Algorithm, to calculate $\gcd(4840, 2101)$.

Short Answer Section Continued (68 points):

15. (3 points) Recall that the amount of pizza varies directly with its diameter squared. Using this knowledge, which is the better deal
- A 12 inch (diameter) pizza that costs \$9, or
 - A 14 inch (diameter) pizza that costs \$12?

You must show all your work that determines the better deal.

16. (3 points) If Alex can paint a room in 5 hours and Betty can paint a room in 8 hours, how long would it take to paint the room together?

17. (3 points) The Brick is having their bi-monthly tent sale where everything is decreased by 40%. Noting that the 10% sales tax is calculated after the discount, how much would a \$500 couch cost?

Short Answer Section Continued (68 points):

18. (5 points) The table below gives the data of an experiment in which people were given pickles to see if they like them:

	Had 1 pickle	Had 2 pickles	Had 3 or more pickles	Total
Likes pickles	78	187	347	612
Hates pickles	292	91	5	388
Total	370	278	352	1000

- (a) What is the probability that a randomly selected person will like pickles?
- (b) What is the probability that a randomly selected person will like pickles and eat at least 3?
- (c) What is the probability that a randomly selected person will hate pickles or eat at least 2?
- (d) What is the probability that a randomly selected person will eat only 1 pickle provided they like pickles?
- (e) What is the probability that a randomly selected person will hate pickles provided they eat at least 2?

Choice Section (32 points): Choose 4 of the following six problems to solve.

19. For each problem, you may encircle **keep** (this tells me you choose this problem) or **ignore** (this tells me you do not choose this problem). One must choose exactly four problems. I will mark the first four problems if ignore is not identified. Show your work.

A) (8 points) 100 people were asked which of the three flavours (vanilla, strawberry, chocolate) of ice cream they enjoyed. These were the results:

KEEP

70 people said they enjoy chocolate ice cream
55 people said they enjoy strawberry ice cream
45 people said they enjoy vanilla ice cream
30 people said they enjoy chocolate and strawberry, but not vanilla
25 people said they enjoy chocolate and vanilla, but not strawberry
15 people said they enjoy strawberry and vanilla, but not chocolate
10 people said they do not enjoy ice cream

IGNORE

- (a) How many people enjoy only chocolate ice cream?
- (b) How many people enjoy only strawberry ice cream?
- (c) How many people enjoy only vanilla ice cream?
- (d) How many people enjoy all three flavours of ice cream?

B) (8 points) Prove that $\sqrt{3}$ is irrational.

KEEP

IGNORE

Choice Section (32 points): Choose 4 problems to solve.

C) (8 points) Use the truth table started below to prove the following logical law:

KEEP

$$[(\neg P \vee Q) \implies R] \iff [R \vee (P \wedge \neg Q)]$$

IGNORE

P	Q	R	$\neg P$	$\neg P \vee Q$	$(\neg P \vee Q) \implies R$	$\neg Q$	$P \wedge \neg Q$	$R \vee (P \wedge \neg Q)$	$[(\neg P \vee Q) \implies R] \iff [R \vee (P \wedge \neg Q)]$
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

D) (8 points) Batgirl, Catwoman and Wonder Woman are going to a red-white-black gala. They are trying to decide what to wear. The colour options are red, white and black for each of shoes, dress and purse. No one is allowed to have the same colour article as anyone else (i.e., two people cannot both have red shoes) nor can one person wears two articles of the same colour. If the following statements are true:

KEEP

IGNORE

- Whoever wears a red dress or a black purse won't wear white shoes.
- Catwoman's shoes match Batgirl's dress and Wonder Woman's purse.
- If Batgirl wears a black dress, then she wears black shoes.
- Wonder Woman does not have a red dress.

Then who wears what to this gala?

Choice Section (32 points): Choose 4 problems to solve.

- E) (8 points) You are taking a large group of students on a field trip. You find out that if you put 12 students on each bus, then 5 students remain, but if you take two buses away then you can divide the students evenly among the remaining buses. How many students are there?

KEEP

IGNORE

- F) (8 points) You have apples in a basket that can hold up to 700 apples. If you arrange the apples in groups of 5, then you have 2 remaining. If you arrange the apples in groups of 9, then you have 4 remaining. Finally, if you arrange the apples in groups of 11, then you have 9 remaining. How many apples are there?

KEEP

IGNORE

Space for extra work.