

Math 103: Calculus for the Social and the Management Sciences
Instructors: Dr. Adam-Christiaan van Roosmalen Section 001
Dr. Dipra Mitra Section 991
Fall 2012, final exam

Family name: _____ **First name:** _____

Student ID: _____ **Section:** _____

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks available:	15	10	5	5	15	5	5	3	6	10	5	16	100
Marks:													

READ THESE INSTRUCTIONS CAREFULLY

1. This examination has 12 questions; there are 10 pages.
2. You have 3 hours to complete this examination.
3. This is a closed book examination, and no notes of any kind are allowed. Cell phones, pagers, or any text storage or communication devices are not permitted.
4. The use of calculators is allowed; however the calculator must meet the guidelines described in the calculator policy of the Department of Mathematics and Statistics.
5. Please show all of your work. It is *your* responsibility to convince us that you know what you are doing! Clarity, completeness, and organization are important.
6. You are expected to define the symbols you introduce and to state any necessary assumptions.
7. Use the backs of the pages if you need extra space.
8. Good luck!

**DO NOT OPEN THIS BOOKLET
UNTIL INSTRUCTED**

1. [15 marks] Find the indicated limit, if it exists.

(a) $\lim_{x \rightarrow 4} \frac{x-4}{x^2-16}$

(b) $\lim_{x \rightarrow 2} \frac{\sqrt{x-1}-1}{x-2}$

(c) $\lim_{x \rightarrow \infty} (1 + x + 2x^2 - x^3)$

2. [10 marks] Find the indicated one-sided limit. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

(a) $\lim_{x \rightarrow 2^-} \frac{1-2x}{x-2}$

(b) $\lim_{x \rightarrow 0^+} \frac{x}{x^2-x}$

3. [5 marks] Determine whether the function $f(x)$ is continuous at $x = 2$. Explain your answer.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ \frac{x^2}{x+2} & \text{if } x > 2 \end{cases}$$

4. [5 marks] Find the equation of the line that is tangent to the graph of the function

$$f(x) = x^4 + x - 1$$

at the point $(-1, -1)$.

5. [15 marks] Differentiate the given functions.

(a) $f(x) = 3x^7 - \frac{1}{x} + 4$

(b) $f(x) = \frac{x^2+1}{x^3-5}$

(c) $f(x) = \sqrt{x^3 + 2x + 3}$

(d) $f(x) = \ln(x^2 + 2)$

(e) $f(x) = xe^{\sqrt{x}}$

6. [5 marks] Find $\frac{dy}{dx}$ by implicit differentiation.

$$y^2 + 2xy + \sqrt{x} = 1$$

7. [5 marks] Air is being pumped into a balloon at a rate of $10 \text{ cm}^3/\text{s}$. How fast is the radius changing (with respect to time) when the radius is 20 cm ? (You can assume that the balloon is a sphere and thus that the volume V of the balloon is $V = \frac{4\pi}{3}r^3$, where r is the radius.)

8. [3 marks] Find $\frac{1}{c} \ln \left[\left(\frac{a}{b} \right)^c \right]$ if $\ln a = 5$ and $\ln b = 3$.

9. [6 marks] Find the absolute maximum and minimum (if any) of the function

$$f(x) = \frac{x}{x+2}$$

on the interval $1 \leq x$, and explain your answer.

10. [10 marks] Find the indicated integrals.

(a) $\int (2x^3 - \sqrt{x} + 4) dx$

(b) $\int xe^{x^2+1} dx$

11. [5 marks] For a certain function $f(x)$, the slope of the tangent line at any point (x,y) on the graph of $f(x)$ is given by the function

$$g(x) = 3x^2 + 1.$$

Find the function $f(x)$ if $f(0) = 1$.

12. [16 marks] Let f be the function $f(x) = \frac{x}{(x-1)^2}$.

- (a) Find the domain.
- (b) Find the x and y intercepts (if the function has any).
- (c) Find the horizontal and vertical asymptotes.
- (d) Find where the function is increasing and decreasing; find all relative extrema.
- (e) Find where the graph is concave upward or concave downward; find all inflection points.

You do not need to plot the graph. You can use that

$$f'(x) = -\frac{x+1}{(x-1)^3} \text{ and } f''(x) = \frac{2x+4}{(x-1)^4}$$