# UNIVERSITY OF REGINA
## Department of Mathematics and Statistics

## Mathematics 110 (Calculus I)
## Final Exam

## 12 December 2015

**Time: 3 hours** **Full Name:**______________________________

**Pages: 13** **Student Number:**____________________________

**Instructor:** (check one)

- ☐ Alison Purdy (001)
- ☐ Gurmail Singh (002)
- ☐ Patrick Maidorn (003)
- ☐ Fernando Szechtman (004)
- ☐ Richard McIntosh (005)
- ☐ Robert Petry (C01/C02)
- ☐ Fotini Labropulu (L01/L02/L03)

### INSTRUCTIONS

1. Full credit is awarded only for well-presented, correct solutions in which all of your work is shown.
2. Your solution and final answer should appear on the right side of the pages in this exam booklet; the left side pages are to be used as scrap paper and are provided for rough work only. If you require more space for your solution than the right side allows, then use the left side (facing) page and indicate on your solution that there is work to be found on the facing page. No other paper is allowed in this exam.
3. Place final answers in blanks when provided.
4. The marks allocated for each question are found to the left of the question.
5. Graphing calculators, tablets, and phones are not permitted.

For instructor use only:

| Page: | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks: | 9 | 13 | 12 | 7 | 7 | 12 | 6 | 10 | 9 | 10 | 5 | 100 |
| Score: | | | | | | | | | | | | |

Marks

(9) 1. Evaluate the given limits:

(a) $\displaystyle\lim_{x\to 4} \frac{2x^2 - 5x - 12}{x^2 - x - 12}$

(b) $\displaystyle\lim_{x\to +\infty} \left(\sqrt{x^2 + 5x - 4} - x\right)$

(c) $\displaystyle\lim_{x\to 0} \frac{2x + 4\sin x}{5x}$

(6) 2. Consider the function $f(x) = x^{\frac{1}{5}}$ on the interval $[1, 32]$.

(a) Verify that the conditions of the Mean Value Theorem are satisfied for the given function and interval, and state the conclusion of the theorem.

(b) Find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem.

(7) 3. (a) State the limit definition of the derivative for the function $f$ at the point $a$.

(b) Use the limit definition in part (a) to calculate $f'(1)$, where $f(x) = \sqrt{3x+1}$.

(12) 4. Find the indicated derivatives. You do not need to simplify your answer.

(a) $f(x) = \dfrac{1}{x^6} + \dfrac{\sqrt{x}}{7} + \pi$ $\quad f'(x) = ?$

(b) $h(\theta) = 5\theta + \theta^2 \cos(\theta)$ $\quad \dfrac{dh}{d\theta} = ?$

(c) $g(z) = \dfrac{(z^2+9)^3}{z^3}$ $\quad g'(z) = ?$

(d) $f(x) = \sqrt{x^3+1}$ $\quad f'(2) = ?$

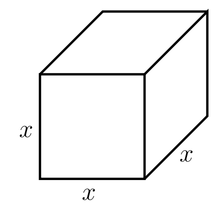
(7) 5. Use implicit differentiation to find an equation of the line tangent to the curve

$$3x^2 + xy - y^3 = 1 \quad \text{at the point} \quad (x, y) = (-1, 1) \; .$$

equation of tangent line:______________________________

(7) 6. A sugar cube dissolving in coffee has edge length $x$ (in mm) and volume $V$ (in mm$^3$).

(a) Assuming the sugar cube maintains the shape of a cube as it dissolves, find an equation relating the rates $\frac{dV}{dt}$ and $\frac{dx}{dt}$.




(b) Because it is the total surface of the cube that interacts with the coffee, the rate at which the volume decreases is proportional to its surface area so that $\frac{dV}{dt} = k(6x^2)$ for some negative constant $k$. Suppose that $k = -1$ in units of mm/s.

i. Show that the rate at which the edge length is decreasing is constant and give the value and units of $\frac{dx}{dt}$.

ii. How long does it take for a sugar cube with initial edge length 10 mm to dissolve?

(12) 7. Consider the function $f(x) = 4x^3 - 3x^2 - 6x$ .

(a) State the domain.

domain:________________

(b) Find all $x$- and $y$-intercepts.

$x$-intercepts:____________________________ $y$-intercepts:________________

(c) Find the values and location of any local (relative) extreme values and state the intervals on which the function is increasing and decreasing.

local max:______________________ local min:______________________

increasing on:___________________ decreasing on:___________________

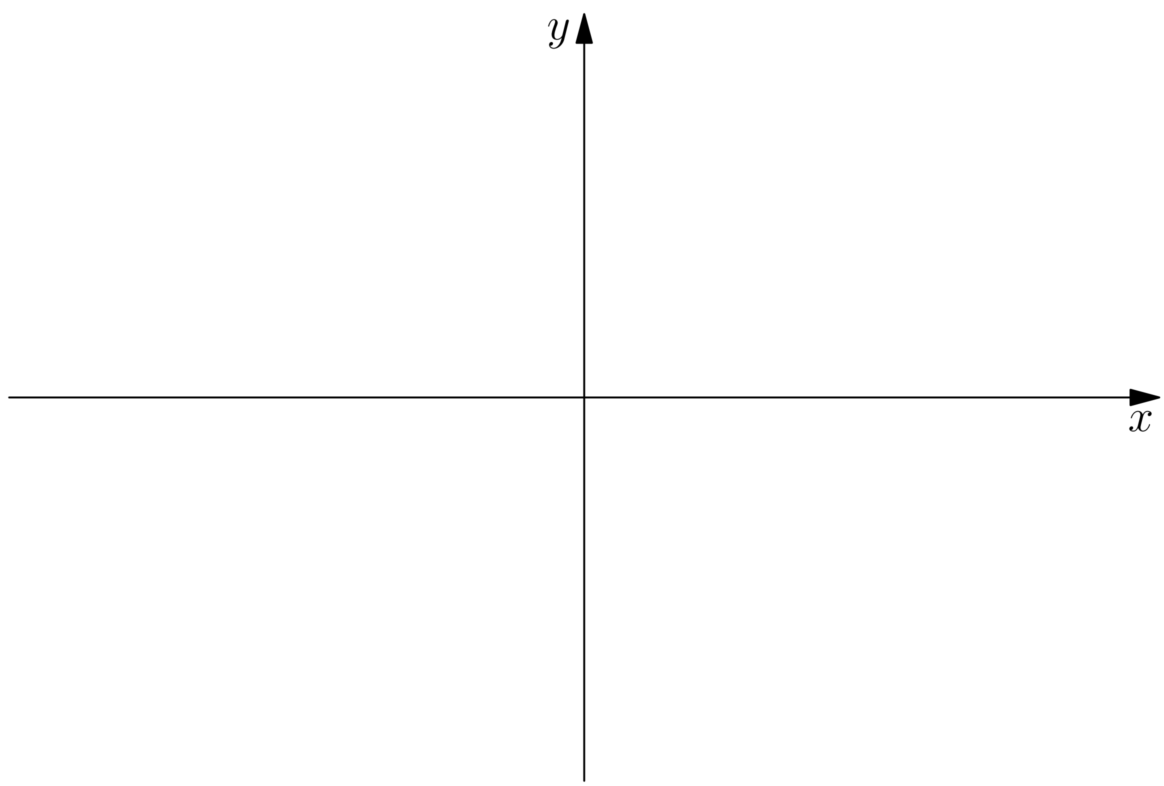
(d) Find any inflection points and state the intervals on which the function is concave up and concave down.

inflection points:________________________

concave up on:________________________

concave down on:________________________

(e) Use your information from (a)-(d) to sketch a graph of the function.

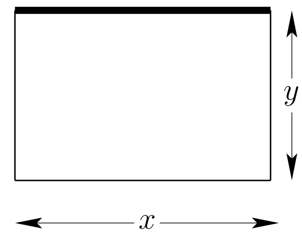


(6) 8. Find and state all horizontal and vertical asymptotes of $f(x) = \dfrac{x^2 + 2x - 3}{1 - x^2}$ .
Show all relevant limits in your work.

vertical asymptotes:__________________

horizontal asymptotes:__________________

(10) 9. A rectangular picture frame encloses an area of 600 $cm^2$. The top edge of the frame is constructed out of heavier material than the other three sides. If the material for the top edge weighs 200 gram/cm and the other three sides are made from material weighing 100 gram/cm, find the dimensions of the frame that would minimize the total weight of the material used.




$x =$ ____________________

$y =$ ____________________

(9) 10. Evaluate each of the following indefinite and definite integrals:

(a) $\int \left(x^2 + \sqrt{x} - 1\right) dx$

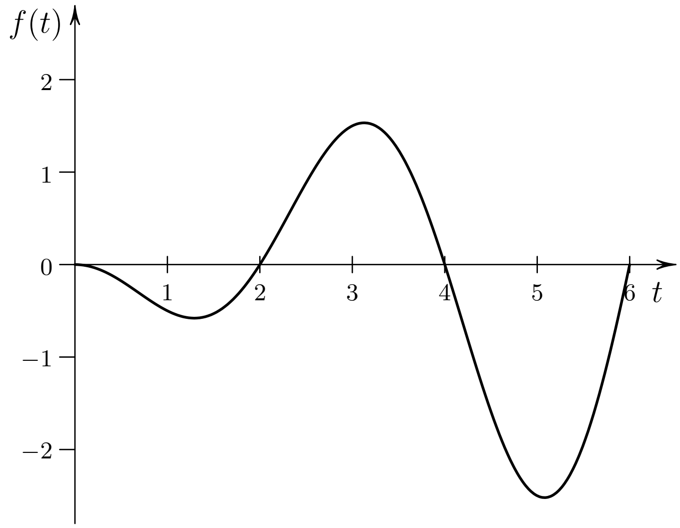
(b) $\int x \sqrt{1 - x^2}\, dx$

(c) $\int_0^{\frac{\pi}{2}} \cos x \, \sin^3 x \, dx$

(10) 11. Find the area enclosed by the curves $y = x^2 + 1$ and $x + y = 1$ . Your solution must include a labeled sketch of the relevant area.

area=____________________

(5) 12. Consider the given graph of the function $f(t)$.




(a) If $A = \int_2^4 f(t)\,dt$ and $B = \int_4^6 f(t)\,dt$ which of the following is correct:

$$A < B, \quad A = B, \quad A > B$$

Clearly justify your answer.

(b) Let $g(x) = \int_0^x f(t)\,dt$. At what $x$-value does $g(x)$ attain its absolute maximum on the interval $[0, 6]$? Clearly justify your answer.

$x =$ ________________