PIMS Distinguished Lecture Series

Dr. Peter Zvengrowski

University of Calgary

Monday
March 17, 2014
3:30 p.m.
Research and
Innovation Centre
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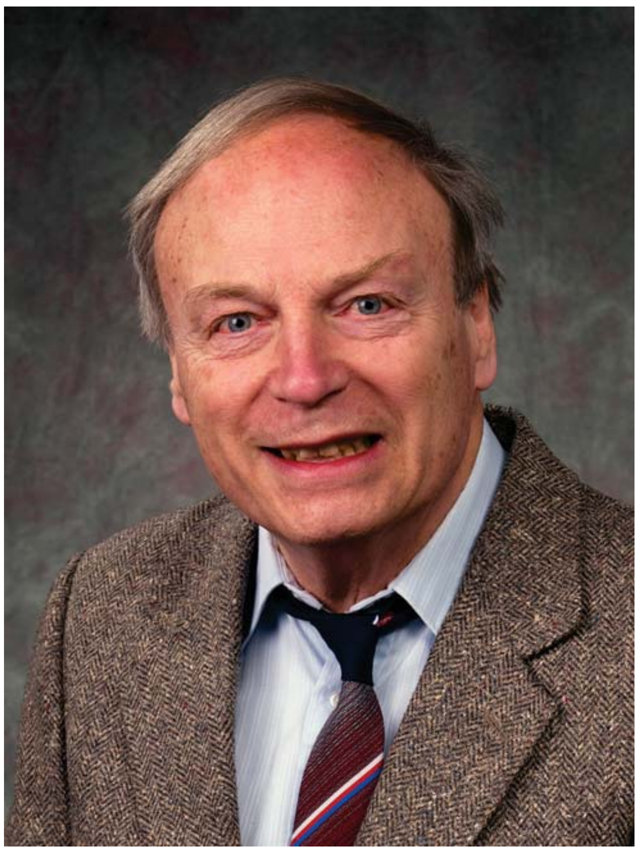

The Riemann Zeta Function, A Horizontal Viewpoint

The first part of this talk will be an historical outline of the Riemann zeta function and related functions such as the Dirichlet *L*-functions, the gamma function Γ, etc., starting from 1350 and into the present century. The closely associated Riemann Hypothesis (RH) and its relation to the distribution of the prime numbers will also be briefly discussed. This will be the main portion of the talk and is non-technical.

The second part of the talk will be shorter and a little more technical, describing recent work by the author and his colleagues, Filip Saidak and Yuri Matiyasevich. Here we will consider monotonicity in the "horizontal direction" for the well known Riemann zeta function $\zeta(s)$, where $s = \sigma + it$, and monotonicity here for any function $f(s)$ means that $|f(s)|$ is monotone increasing or monotone decreasing as σ increases, i.e. the modulus $|f(s)|$ is monotone in the horizontal direction. A similar result will be outlined for the two related functions $\eta(s)$ (the Euler-Dedekind eta function) and $\xi(s)$ (the Riemann ξ function). Here it will be shown that all three have monotone decreasing modulus for $\sigma < 0$ and $|t| > 8$, and that for any of the three functions the extension of this monotonicity result to the larger region $\sigma < ½$; $|t| > 8$, is equivalent to RH. An inequality relating the monotonicity of all three functions will be given, and very similar results for Dirichlet *L*-functions will be mentioned.

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Mathematics and Statistics

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