

- URMS -  
Undergraduate Research  
Seminar in Mathematics and  
Statistics

Shaun Fallat

On the multiplicativity of the  
determinant: Where are  
Cauchy & Binet?

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**Abstract:** In a first course on Linear Algebra, students are asked to learn about the determinant of a matrix, as a tool used to calculate the eigenvalues of a matrix. Typically, just some basic properties of the determinant are discussed, leaving the wrong first impression about this important function. Further, students often do not pick up on the fact that viewing eigenvectors as invariant subspaces helps to truly synthesize the concepts of eigenvalues and diagonalization.

Students are usually shown that  $\det(AB) = \det A \cdot \det(B)$ , for  $n \times n$  matrices  $A$  and  $B$  – sometimes with a short sketch of the proof. One issue to consider, is what if  $A$  and  $B$  are rectangular, but  $AB$  is square, how then can we compute  $\det(AB)$  in terms of  $A$  and  $B$ ?

I will demonstrate how to do this, and I will show that the equation  $\det(AB) = \det A \cdot \det(B)$  and conventional matrix multiplication (also defined for rectangular matrices), are both simple instances of a much more universal and beautiful identity on the determinant that, sadly, is not commonly taught at the undergraduate level...

