COLLOQUIUM

Douglas Farenick
University of Regina

Algebraic properties and isometries of Toepltiz operators and matrices

Mathematics and Statistics

Date: Friday, Mar 20, 2015; Time: 3:30 - 4:30 PM; Room: RIC 209

Abstract: If φ is an essentially bounded 2π-periodic Lebesgue measurable function $\mathbb{R} \to \mathbb{C}$, then φ admits a Fourier series decomposition of the form $\varphi = \sum_{n\in\mathbb{Z}} \alpha_n e_n$, where $e_n(t) = e^{2\pi int}$, $n \in \mathbb{Z}$. Each such φ induces a bounded linear operator T_φ on the Hardy space $H^2(\mathbb{T})$ of $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ such that the matrix of T_φ with respect to the orthonormal basis $\{e_n\}_{n=0}^{\infty}$ of $H^2(\mathbb{T})$ is given by

$$T_\varphi = \begin{pmatrix} \alpha_0 & \alpha_{-1} & \alpha_{-2} & \alpha_{-3} & \cdots \\ \alpha_1 & \alpha_0 & \alpha_{-1} & \alpha_{-2} & \ddots \\ \alpha_2 & \alpha_1 & \alpha_0 & \alpha_{-1} & \ddots \\ \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}.$$

The matrix T_φ above is called a *Toeplitz matrix*. Truncating T_φ to the leading $n \times n$ principal submatrix leads to a *finite Toeplitz matrix* A_φ:

$$A_\varphi = \begin{pmatrix} \alpha_0 & \alpha_{-1} & \cdots & \alpha_{1-n} \\ \alpha_1 & \alpha_0 & \ddots & \ddots \\ \ddots & \ddots & \ddots & \alpha_{-1} \\ \alpha_{n-1} & \ddots & \alpha_1 & \alpha_0 \end{pmatrix}.$$

Toeplitz matrices clearly exhibit a great deal of special structure. In this lecture I will describe algebraic features of the linear space of Toeplitz matrices pertaining to normality and hyponormality. I will also discuss recent work with M. Mastnak and A. Popov on the structure of isometries of the algebra of finite upper-triangular Toeplitz matrices.

University of Regina