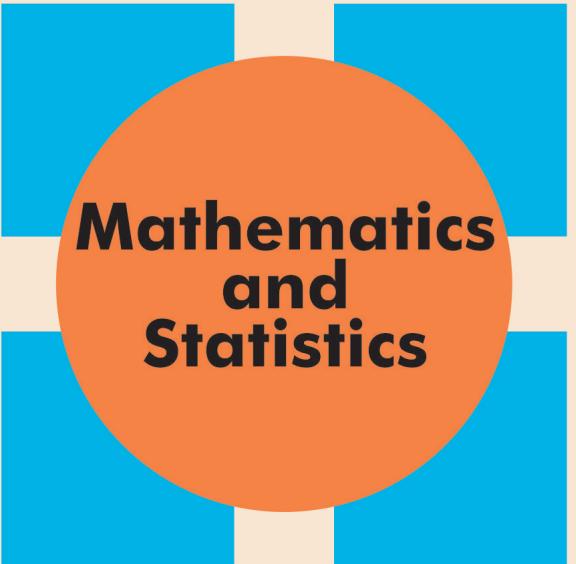


# COLLOQUIUM

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## Random walks and percolation



Mathematics  
and  
Statistics

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**Abstract:** Around 1960 de Giorgi, Moser and Nash proved regularity for the heat equation associated with divergence form PDE:

$$\frac{\partial u}{\partial t} = Lu, \quad u = u(x, t), x \in \mathbf{R}^d, t > 0. \quad (1)$$

Here  $L = \nabla a \nabla$ , where  $a = a_{ij}(x)$  is uniformly elliptic. In the special case when  $a = \rho(x)I$ , (1) describes heat flow in a medium of varying conductivity  $\rho$ , and uniformly elliptic means that  $\rho$  is bounded away from 0. If on the other hand  $Z = \{x : \rho(x) = 0\} \neq \emptyset$ , then the zero regions can form barriers, and the behaviour of solutions to (1) will depend on the detailed geometry of  $Z$ .

One can also consider discrete approximations to (1) which have regions of zero conductivity. One model which leads to regions of this kind is percolation on the Euclidean lattice  $\mathbf{Z}^d$ , which was introduced by Broadbent and Hammersley in 1957. Let  $p$  be a fixed probability between 0 and 1. Each bond in  $\mathbf{Z}^d$  is retained with probability  $p$ , and removed with probability  $1 - p$ , independently of all the others. If  $p$  is larger than some critical value  $p = p_c(d)$  then the resulting graph has a unique infinite connected component  $C$ . De Gennes in 1976 called the random walk on  $C$  the 'the ant in the labyrinth'. The problem is related to the heat equation, since if  $p(n, x, y)$  is the probability that a random walker ('the ant'), starting at  $x$ , is at  $y$  at time  $n$ , then  $p$  satisfies a discrete version of (1).

The PDE techniques of Nash and Moser have proved very useful in these contexts. I will discuss recent progress on these models.