Mathematics and Statistics

COLLOQUIUM

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Random walks and percolation

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Abstract: Around 1960 de Giorgi, Moser and Nash proved regularity for the heat equation associated with divergence form PDE:

$$\frac{\partial u}{\partial t} = Lu, \quad u = u(x,t), x \in \mathbf{R^d}, t > 0. \tag{1}$$

Here $L = \nabla a \nabla$, where $a = a_{ij}(x)$ is uniformly elliptic. In the special case when $a = \rho(x) I$, (1) describes heat flow in a medium of varying conductivity ρ, and uniformly elliptic means that ρ is bounded away from 0. If on the other hand $Z = \{x : \rho(x) = 0\} \neq \emptyset$, then the zero regions can form barriers, and the behaviour of solutions to (1) will depend on the detailed geometry of Z.

One can also consider discrete approximations to (1) which have regions of zero conductivity. One model which leads to regions of this kind is percolation on the Euclidean lattice $\mathbf{Z}^d$, which was introduced by Broadbent and Hammersley in 1957. Let p be a fixed probability between 0 and 1. Each bond in $\mathbf{Z}^d$ is retained with probability p, and removed with probability $1-p$, independently of all the others. If p is larger than some critical value $p = p_c(d)$ then the resulting graph has a unique infinite connected component C. De Gennes in 1976 called the random walk on C the 'the ant in the labyrinth'. The problem is related to the heat equation, since if $p(n,x,y)$ is the probability that a random walker ('the ant'), starting at x, is at y at time n, then p satisfies a discrete version of (1).

The PDE techniques of Nash and Moser have proved very useful in these contexts. I will discuss recent progress on these models.

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