

Topics in Geometry Seminar

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Almost Complex Manifolds III

Date: March 13, 2017

Time: 11:30 - 1:00 PM

Room: Math & Stat Lounge 307.20

Abstract:

My purpose in the first few talks is to review of the fundamental concepts on which the remainder of the seminar rests. Namely, we will discuss basics of smooth manifolds, tangent bundles and almost complex structures, as well as the definition of analytic functions in \mathbb{C}^n and some of their basic properties in order to introduce the definition of complex manifolds. Since every complex analytic function is smooth (actually real analytic) in the real sense then every complex manifold is smooth (by identifying \mathbb{C}^n with \mathbb{R}^{2n}). In contrast, not every $2n$ -dimensional smooth manifold is complex, in fact not even an almost complex manifold e.g \mathbb{S}^4 . We will discuss integrability of almost complex structures and we will see that \mathbb{S}^6 admits an almost complex structure and we will hopefully show that this structure is not integrable i.e. not complex.

In this seminar we will be focusing in Atiyah's last paper where he claimed a proof that \mathbb{S}^6 has no complex structure. Hence, it turns out that \mathbb{S}^2 is the only sphere which is a complex manifold and \mathbb{S}^6 is the only sphere which is an almost complex manifold but not complex. The proof of \mathbb{S}^4 , \mathbb{S}^{2n} , $n > 3$ are not almost complex manifolds is not trivial and requires some characteristic classes theory which we may come back at the end of the seminar and discuss it.