COLLOQUIUM

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Operator-valued Hardy spaces: characterizations, atomic decompositions, and applications to pseudo-differential operators

Mathematics and Statistics

Friday, November 03, 2017; 3:30 - 4:30 PM; ED 314

Abstract: In 2006, Tao Mei developed a remarkable theory of operator-valued Hardy spaces on $\mathbb{R}^d$, defined by the Littlewood-Paley g-function or Lusin area integral function associated to the Poisson kernel. The H_1-BMO duality, Calderön-Zygmund theory and interpolations were obtained in Mei's paper. These spaces are shown to be very useful for many aspects of noncommutative harmonic analysis. I am going to talk about some development on these Hardy spaces in our recent papers.

-Characterizations. We show that, as in the scalar case, the Poisson kernel can be replaced by any good enough convolution kernel. This is also true for $I^\alpha P$ as soon as $\alpha > 0$, where I^α is the α order Riesz potential, and P is the Poisson kernel. This last characterization seems new even in the classical case.

-Local versions. We define local Hardy spaces by using the truncated versions of the Littlewood-Paley g-function or Lusin area integral function. These spaces enjoy almost the same properties as Mei's Hardy spaces. We can even refine the smoothness of the atomic decomposition.

-Pseudo-differential operators. We define operator-valued pseudo differential operators, by replacing the absolute values in the scalar case with the operator norms. These operators are not bounded on Hardy spaces in general, but bounded on their local versions. Applying the smooth atomic decomposition, we prove the boundedness of pseudo-differential operators with regular symbols on local Hardy spaces.

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