COLLOQUIUM

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The Feasibility Problem for Integral Table Algebras

Mathematics and Statistics

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Abstract: Integral table algebras are a finite-dimensional associative $\mathbb{C}$-algebras with involution $*$, that have a distinguished basis $\mathbf{B}$ that (i) contains 1, (ii) is $*$-invariant, (iii) produces nonnegative integer structure constants, and (iv) satisfies the following *pseudo-inverse* condition: for all $b \in \mathbf{B}$, b^* is the only element $c \in \mathbf{B}$ for which the coefficient of 1 in bc is not 0.

The most interesting examples of integral table algebra bases are finite groups G and the collections S of adjacency matrices in a finite association scheme. In fact, it turns out to be an extremely interesting problem (both theoretically and computationally) to determine when an integral table algebra basis can be represented as the collection of adjacency matrices of an association scheme.

Computationally, this feasibility problem can be presented in a manner comparable to permutation puzzle games like Rubik's cube or Hex. We demonstate our implementation of simple random and backtracking attacks which has been successful in all cases of the feasibility problem with order up to 30.

Theoretically, this problem is essential for the search for counterexamples to the cyclotomic eigenvalue conjecture, which has stalemated extension of rational representation theory of finite groups to the setting of association schemes for the past 40 years. We will demonstrate our implementation of rational representation theory algorithms for finite groups that is available in the GAP package `wedderga`.

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