HONOURS SEMINAR

Johnna Parenteau

Zero forcing, Hidden Triangles, and Forts

Supervised by Dr. Shaun Fallat

November 18, 2021
9:30 am

Abstract:

Zero forcing is a combinatorial game played on a graph G where the goal is to fill the unfilled vertices of a graph at minimal cost, denoted as $Z(G)$, using two specified actions: (1) at any point, a vertex can be filled for a cost of one token, and (2) at no cost, the player can apply the filling rule. This number, $Z(G)$, is used to study the maximum nullity or minimum rank of the family of symmetric matrices associated to G. It is shown that in order to find the vertices that contribute to $Z(G)$, a lower triangular submatrix, called a *hidden triangle*, can be found for each matrix associated to G, and a subset of vertices of G, called *forts*, can graphically predict their locations. For a variation of zero forcing, namely the q-analogue of zero forcing, a similar combinatorial game is presented for the graphs associated to the family of symmetric matrices with precisely $q > 0$ negative eigenvalues. This game has the additional rule that (3) any filled vertices are cut from the graph, and at most $q + 1$ unfilled components are reattached through the discretion of an oracle and filled when applicable at minimum cost, denoted $Z_q(G)$. From here, a reproof of P. Nylen's work on the support of null vectors [*Linear Algebra and its Applications*, Vol. 279, pg.151-161] is given as motivation for finding *q-forts* associated to $Z_q(G)$.

University of Regina

Mathematics and Statistics